Correctness Proof of the 3NF Decomposition Algorithm

Let G be a minimal cover of F. Let $X \to A$ be any FD in G. Let T be a table with schema $X \cup \{A\}$. We will prove that T is in 3NF.

Lemma 1. *X is a candidate key of T.*

Proof. Suppose that X is not a candidate key of T. Then, there exists $Y \subset X$ such that $Y \to A$, namely, $Y \to A$ can be derived from G. As shown next, this will lead to the contradiction that G is not a minimal cover.

Let H be the set of FDs in G other than $X \to A$ (i.e., $G = H \cup \{X \to A\}$). Define $G' = H \cup \{Y \to A\}$. Next, we will show that $G^+ = G'^+$, namely, G can still be simplified and hence, cannot be minimal.

Claim 1: $G'^+ \subseteq G^+$. That is, if a FD can be derived from G', we can also derive it from G. This is true because (as mentioned earlier) we can derive $Y \to A$ from G, and hence, the entire G' from G.

Claim 2: $G^+ \subseteq G'^+$. That is, if a FD can be derived from G, we can also derive it from G'. This is true because we can derive $X \to A$ from the FD $Y \to A$ in G' through transitivity: $X \to Y \to A$. In other words, we can derive the entire G from G'. □

Now we are ready to establish:

Lemma 2. *T is in 3NF.*

Proof. Now suppose that T is not in 3NF. Let $Y \to B$ be a 3NF-violating FD. First observe that B must be A; otherwise, by the fact that B is an attribute in T, we know $B \in X$, but in this case, B is in a candidate key of T, and hence, cannot be a 3NF-violating FD.

Given that $B = A$, we know $Y \neq X$, and hence, $Y \subset X$. In other words, we have identified a FD $Y \to A$ which can be derived from G, By the same argument as in the proof of Lemma 1, we can show that G is not minimal, and therefore, a contradiction. □