Graph Storage: Adjacency Lists

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Undirected Graphs

Let us first recall:

An **undirected graph** is a pair of (V, E) where:

- V is a set of elements, each of which called a **node** (or **vertex**).
- E is a set of unordered pairs $\{u, v\}$ (each called an **edge**) where u and v are nodes.

If $\{u, v\}$ belongs to E, then we say that u is a **neighbor** of v, and conversely, v is a **neighbor** of u.

Directed Graphs

An **directed graph** is a pair of (V, E) where:

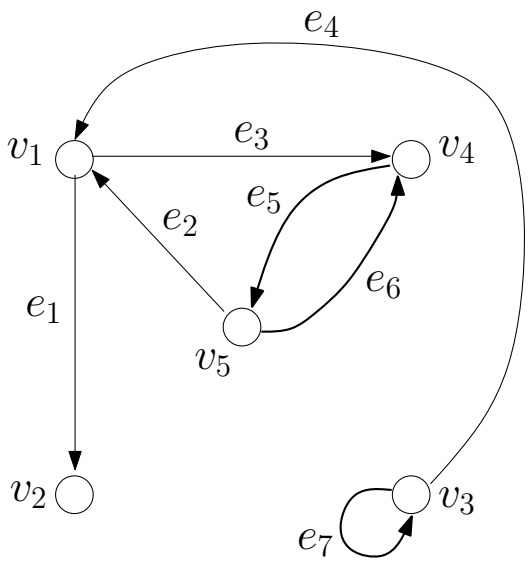
- V is a set of elements, each of which called a **node** (or **vertex**).
- E is a set of pairs (u, v) where u and v are nodes in V.

Each element $(u, v) \in E$ is a **directed edge**.

- More specifically, it is an **outgoing** edge of u and an **incoming** edge of v.

Accordingly, v is an **out-neighbor** of u and u is an **in-neighbor** of v.

Example

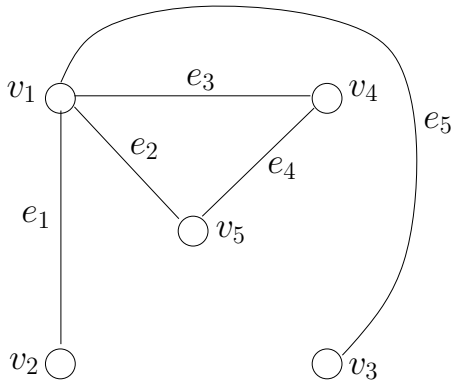


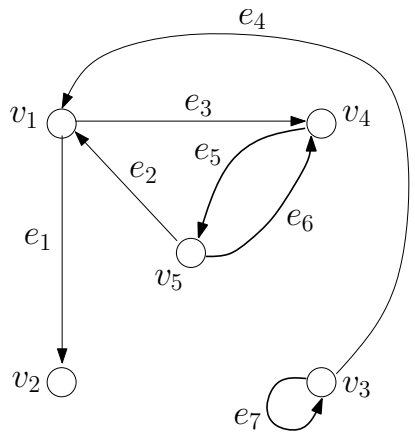
This is a directed graph (V, E) where there are 5 vertices $v_1, v_2, ..., v_5$, and 7 edges $e_1, e_2, ..., e_7$. Edge e_6 is an outgoing edge of v_5 and an incoming edge of v_4.

Degrees

- In an undirected graph, the **degree** of a vertex u is the number of edges incident on u.
- In a directed graph, the **out-degree** of a vertex u is the number its outgoing edges, and the **in-degree** of u is the number of its incoming edges.

Example



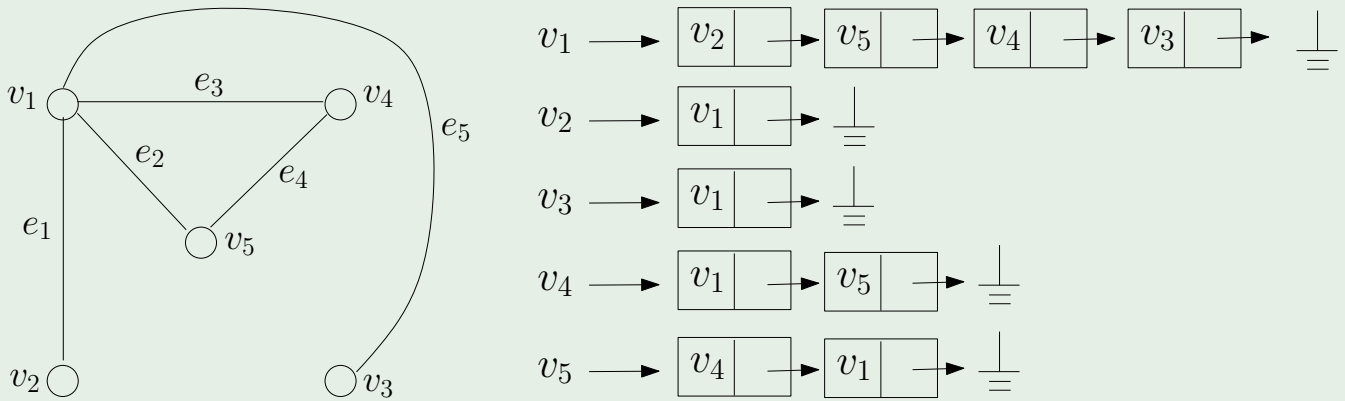


In the left graph, the degree of v_5 is 2. In the right graph, the out-degree of v_3 is 2 and its in-degree is 1.

Adjacency List — Undirected Graphs

Each vertex $u \in V$ is associated with a linked list that enumerates all the neighbors of u.

Example

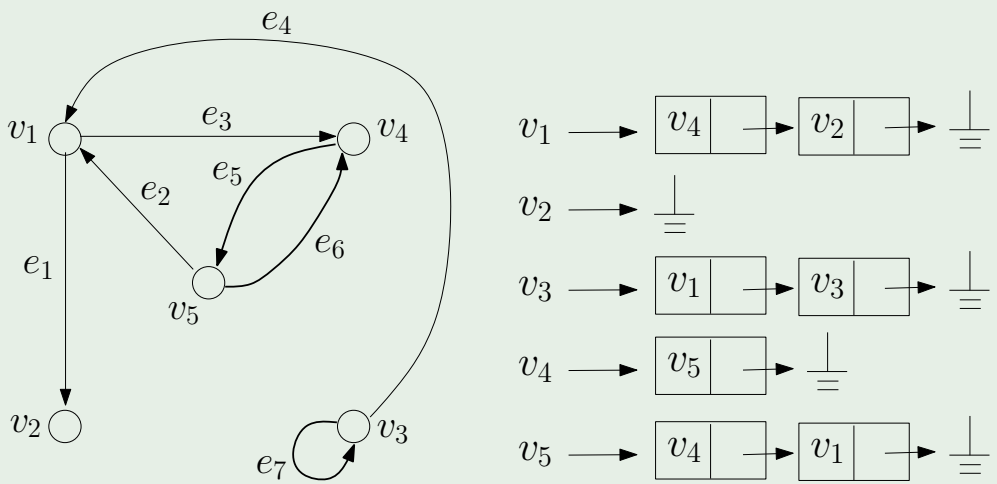


Space $= O(|V| + |E|)$.

Adjacency List — Directed Graphs

Each vertex $u \in V$ is associated with a linked list that enumerates all the out-neighbors of u.

Example



Space $= O(|V| + |E|)$.