ENGG1410-F Tutorial 3

Shangqi Lu

Department of Computer Science and Engineering
The Chinese University of Hong Kong

Problem 1

Suppose the following linear system $Ax = b$ has at least one non-zero solution (in which at least one of x_1, x_2 and x_3 is non-zero). Calculate the possible values of λ.

$$\begin{cases} (1+\lambda)x_1 + x_2 + x_3 = 0 \\ x_1 + (1+\lambda)x_2 + x_3 = 0 \\ x_1 + x_2 + (1+\lambda)x_3 = 0 \end{cases}$$

Note: regardless of λ, $x_1 = x_2 = x_3 = 0$ is always a solution.

Solution

As mentioned, $x_1 = x_2 = x_3 = 0$ is always a solution. If the system has at least one non-zero solution, it means that the system has more than one solution.

According to the consistency criterion theorem, $det(A) = 0$.

$$\begin{vmatrix} 1+\lambda & 1 & 1 \\ 1 & 1+\lambda & 1 \\ 1 & 1 & 1+\lambda \end{vmatrix} = 0$$

Solving the equation, we know that $\lambda = 0$ or $\lambda = -3$.

Problem 2

Calculate the determinant of the following matrix.

$$\begin{bmatrix} 0 & 0 & \cdots & 0 & 1 & 0 \\ 0 & 0 & \cdots & 2 & 0 & 0 \\ \vdots & \vdots & \ldots & \vdots & \vdots & \vdots \\ 2018 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 & 2019 \end{bmatrix}$$

Solution

Let A be an $n \times n$ matrix. Denote by $r_i (i = 1, 2, ..., n)$ the i-th row vectors of A.

Recall that the determinant of A:

- should be multiplied by 1 after switching two rows of A.
- should be multiplied by c after multiplying all numbers of a row of A by the same non-zero value c.
- has no change after updating row r_i to $r_i + r_j$.

Solution

Transform the given matrix into the following diagonal matrix by only using the operation of switching two columns :

$$\begin{bmatrix} 1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 2 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 2018 & 0 \\ 0 & 0 & \cdots & 0 & 0 & 2019 \end{bmatrix}$$

You can verify that there is an odd number of such operations (think: why). The answer is $-2019!$ (factorial of 2019).