# ENGG1410-F: Quiz 1

**Name:**                       **Student ID:**

Write all your answers on this sheet, and use the back if necessary.

**Problem 1 (60%).** Consider the system of equations below:

$$
\begin{aligned}
10x + 4y - 2z &= -4 \\
-3w - x + y + 2z &= 2 \\
w + x + y &= 6 \\
8w - 3x + 16y - 10z &= 4
\end{aligned}
$$

Answer the following questions:

(a) Give the augmented matrix $\tilde{A}$ of the system.

(b) Convert $\tilde{A}$ into row echelon form (you need to show the detailed steps).

(c) Find the rank of $\tilde{A}$.

(d) Apply Gauss elimination to solve the system (you need to show the detailed steps).

**Answer:** (a)

$$
\tilde{A} \;=\;
\left[
\begin{array}{cccc|c}
0 & 10 & 4 & -2 & -4 \\
-3 & -1 & 1 & 2 & 2 \\
1 & 1 & 1 & 0 & 6 \\
8 & -3 & 16 & -10 & 4
\end{array}
\right]
$$

(b)

$$
\tilde{A} \Rightarrow
\left[
\begin{array}{cccc|c}
1 & 1 & 1 & 0 & 6 \\
-3 & -1 & 1 & 2 & 2 \\
8 & -3 & 16 & -10 & 4 \\
0 & 10 & 4 & -2 & -4
\end{array}
\right]
\Rightarrow
\left[
\begin{array}{cccc|c}
1 & 1 & 1 & 0 & 6 \\
0 & 2 & 4 & 2 & 20 \\
0 & -11 & 8 & -10 & -44 \\
0 & 10 & 4 & -2 & -4
\end{array}
\right]
$$

$$
\Rightarrow
\left[
\begin{array}{cccc|c}
1 & 1 & 1 & 0 & 6 \\
0 & 1 & 2 & 1 & 10 \\
0 & -1 & 12 & -12 & -48 \\
0 & 10 & 4 & -2 & -4
\end{array}
\right]
\Rightarrow
\left[
\begin{array}{cccc|c}
1 & 1 & 1 & 0 & 6 \\
0 & 1 & 2 & 1 & 10 \\
0 & 0 & 14 & -11 & -38 \\
0 & 0 & -16 & -12 & -104
\end{array}
\right]
$$

$$
\Rightarrow
\left[
\begin{array}{cccc|c}
1 & 1 & 1 & 0 & 6 \\
0 & 1 & 2 & 1 & 10 \\
0 & 0 & -2 & -23 & -142 \\
0 & 0 & 4 & 3 & 26
\end{array}
\right]
\Rightarrow
\left[
\begin{array}{cccc|c}
1 & 1 & 1 & 0 & 6 \\
0 & 1 & 2 & 1 & 10 \\
0 & 0 & -2 & -23 & -142 \\
0 & 0 & 0 & -43 & -258
\end{array}
\right]
$$

(c) 4

(d) From $-43z = -258$, we get $z = 6$. From $(-2)y - 23 \times 6 = -142$, we get $y = 2$. From $x + 2y + z = 10$, we get $x = 0$. From $w + x + y = 6$, we get $w = 4$.

**Problem 2 (20%).** Find the rank of the following matrix:

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 \end{bmatrix}$$

**Answer.**

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -4 & -6 & -8 \\ 0 & -6 & -9 & -12 \\ 0 & -8 & -12 & -16 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -2 & -3 & -4 \\ 0 & -6 & -9 & -12 \\ 0 & -8 & -12 & -16 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & -2 & -3 & -4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Therefore, the rank is 2.

**Problem 3 (20%).** Compute the following determinant:

$$\begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}$$

**Answer.**

$$\begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix} = a \cdot \begin{vmatrix} a & b \\ c & a \end{vmatrix} - b \cdot \begin{vmatrix} c & b \\ b & a \end{vmatrix} + c \cdot \begin{vmatrix} c & a \\ b & c \end{vmatrix}$$

$$= a(a^2 - bc) - b(ac - b^2) + c(c^2 - ab)$$
$$= a^3 - abc - abc + b^3 + c^3 - abc$$
$$= a^3 + b^3 + c^3 - 3abc.$$