Exercises: Surfaces

Problem 1. Consider the sphere $(x-1)^2 + (y-2)^2 + z^2 = 6$.

1. Give a normal vector of the sphere at point $(2, 2+\sqrt{2}, \sqrt{3})$.
2. Give the equation of the tangent plane at point $(2, 2+\sqrt{2}, \sqrt{3})$.

Problem 2. As before, consider the sphere $(x-1)^2 + (y-2)^2 + z^2 = 6$.

1. Let C_1 be the curve on the sphere satisfying $x = 2$. Give a tangent vector $\boldsymbol{v}_1$ of C_1 at point $(2, 2+\sqrt{2}, \sqrt{3})$.
2. Let C_2 be the curve on the sphere satisfying $y = 2+\sqrt{2}$. Give a tangent vector $\boldsymbol{v}_2$ of C_2 at point $(2, 2+\sqrt{2}, \sqrt{3})$.
3. Compute $\boldsymbol{v}_1 \times \boldsymbol{v}_2$.

Problem 3. Sphere $(x-1)^2 + (y-2)^2 + z^2 = 6$ can also be represented in the parametric form:

$$\begin{aligned} x(u,v) &= 1 + \sqrt{6}\cos(u) \\ y(u,v) &= 2 + \sqrt{6}\sin(u)\cos(v) \\ z(u,v) &= \sqrt{6}\sin(u)\sin(v) \end{aligned}$$

By fixing v to the value satisfying $\cos(v) = \sqrt{2/5}$ and $\sin(v) = \sqrt{3/5}$, from the above we get a curve C on the sphere that passes point $p = (2, 2+\sqrt{2}, \sqrt{3})$. Give a tangent vector of C at the point.

Problem 4. This problem is designed to show you how to use gradient to compute the normal vector of a tangle line in 2d space. Consider the circle $(x-1)^2 + (y-2)^2 = 5$. Give a vector whose direction is perpendicular to the tangent line of the circle at point $(2, 4)$.