Exercises: Matrix Rank

Problem 1. Calculate the rank of the following matrix:

$$\begin{bmatrix} 0 & 16 & 8 & 4 \\ 2 & 4 & 8 & 16 \\ 16 & 8 & 4 & 2 \\ 4 & 8 & 16 & 2 \end{bmatrix}$$

Problem 2. Calculate the rank of the following matrix:

$$\begin{bmatrix} 4 & -6 & 0 \\ -6 & 0 & 1 \\ 0 & 9 & -1 \\ 0 & 1 & 4 \end{bmatrix}$$

Problem 3. Judge whether the following vectors are linearly independent.

$$[3, 0, 1, 2]$$
$$[6, 1, 0, 0]$$
$$[12, 1, 2, 4]$$
$$[6, 0, 2, 4]$$
$$[9, 0, 1, 2]$$

If they are not, find the largest number of linearly independent vectors among them.

Problem 4. Prove: if $\boldsymbol{A}$ is not square, then either the row vectors or the column vectors are linearly dependent.

Problem 5. Let S be an arbitrary set of 3×1 vectors. Prove that there are at most 3 linearly independent vectors in S.

Problem 6 (Hard). Prove: $rank(\boldsymbol{AB}) \leq rank\boldsymbol{A}$.

Problem 7 (Very Hard). Prove: $rank(\boldsymbol{A} + \boldsymbol{B}) \leq rank\ \boldsymbol{A} + rank\ \boldsymbol{B}$.