Exercises for CSCI5010

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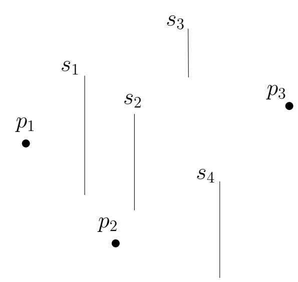
Problem 1. You are given the coordinates of three points in $\mathbb{R}^2$. Describe an algorithm to calculate in constant time the area of the triangle that has the three points as vertices. You should note that $\sqrt{x}$ is not an atomic operation of the real-RAM model.

Problem 2. Let S be a set of n vertical line segments in $\mathbb{R}^2$ (i.e., each segment has the form $x \times [y_1, y_2]$). Also, let P be a set of m points in $\mathbb{R}^2$. For each segment $s \in S$, we want to output a pair (s, p) where p is the first point in P that is hit by s if s moves left; if p does not exist, output (s, nil). For instance, in the following example, you should output $\{(s_1, p_1), (s_2, p_1), (s_3, nil), (s_4, p_2)\}$.

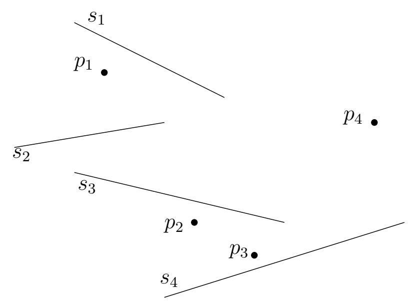


Use the planesweep approach to design an algorithm to solve the above problem in $O(n \log n + m \log m)$ time, subject to the constraint that your algorithm should sweep a horizontal line from $y = -\infty$ to $y = \infty$. You may assume that no two segments in S share the same x-coordinate.

Problem 3 (Range Max). Let S be a set of n real numbers. Each number $v \in S$ is associated with a real valued *weight*. Given a range $[x, y]$, a query returns an element in $S \cap [x, y]$ with the maximum weight. For example, if $S = \{(1, 15), (3, 7), (7, 12), (10, 9)\}$, where each pair has the form $(v, weight(v))$. Then, a query with range $[2, 15]$ returns $(7, 12)$. Design a data structure to answer such queries in $O(\log n)$ time. Your structure should also support insertions and deletions in $O(\log n)$ time.

Problem 4. Consider again Problem 1. Design another planesweep algorithm to solve the above problem in $O(n \log n + m \log m)$ time. This time, your algorithm must sweep a vertical line from $x = -\infty$ to $x = \infty$. You may assume that no two points in P have the same y-coordinate.

Problem 5. Let S be a set of n disjoint line segments in $\mathbb{R}^2$ (these segments can have arbitrary "slopes"), and P be a set of m points in $\mathbb{R}^2$ such that no point in P falls on any segment in S. For each point $p \in P$, we want to output the segment $s \in S$ that is immediately above p, namely, s is the first segment hit by p if p moves up. For instance, in the following example, you should output $\{(p_1, s_1), (p_2, s_1), (p_3, s_1), (p_4, nil)\}$. Design an algorithm to achieve the purpose in $O(n \log n + m \log m)$ time.



Problem 6 (Rotating Sweep; Exercise 2.14 from textbook). Let S be a set of n disjoint line segments in the plane, and let p be a point not on any of the line segments in S. We want to determine all line segments of S that p can see, that is, all line segments of S that contain some point q so the segment pq does not intersect any segment in S (except at q, of course). Give an $O(n \log n)$ time algorithm to solve the problem. For example, in the following figure, you should output all segments but s_4 and s_6.