CMSC5724: Exercise List 2

Answer the following questions based on the training set below

A	B	C	**class**
0	2	1	$+$
1	2	2	$+$
1	1	0	$+$
2	0	2	$+$
1	1	1	$+$
2	2	2	$-$
0	1	0	$-$
0	0	1	$-$
2	1	0	$-$
1	0	0	$-$

Problem 1. Estimate $\mathbf{Pr}[class = +]$ and $\mathbf{Pr}[C = 2 \mid class = +]$.

Answer. Since 5 out of 10 records belong to the class $+$, we estimate $\mathbf{Pr}[class = +]$ to be $1/2$. Among the 5 records of class $+$, 2 of them have $C = 2$. Hence, we estimate $\mathbf{Pr}[C = 2 \mid class = +]$ to be $2/5$.

Problem 2. Let us make the following *conditional independence assumption*: conditioned on a specific class, attributes A, B, C are independent. Estimate $\mathbf{Pr}[A = 1, B = 2 \mid class = +]$.

Answer. Under the assumption we have:

$$\mathbf{Pr}[A = 1, B = 2 \mid class = +] = \mathbf{Pr}[A = 1 \mid class = +] \cdot \mathbf{Pr}[B = 2 \mid class = +]$$

We estimate $\mathbf{Pr}[A = 1 \mid class = +] = 3/5$ and $\mathbf{Pr}[B = 2 \mid class = +] = 2/5$. Therefore, our estimate of $\mathbf{Pr}[A = 1, B = 2 \mid class = +]$ is $6/25$.

Problem 3. Under the conditional independence assumption, decide the larger probability between $\mathbf{Pr}[class = + \mid A = 1, B = 2, C = 0]$ and $\mathbf{Pr}[class = - \mid A = 1, B = 2, C = 0]$.

Answer. By Bayes Theorem

$$\mathbf{Pr}[class = + \mid A = 1, B = 2, C = 0] = \frac{\mathbf{Pr}[A = 1, B = 2, C = 0 \mid class = +] \cdot \mathbf{Pr}[class = +]}{\mathbf{Pr}[A = 1, B = 2, C = 0]}$$

and

$$\mathbf{Pr}[class = - \mid A = 1, B = 2, C = 0] = \frac{\mathbf{Pr}[A = 1, B = 2, C = 0 \mid class = -] \cdot \mathbf{Pr}[class = -]}{\mathbf{Pr}[A = 1, B = 2, C = 0]}$$

To compare the two, we only need to estimate the numerators of the above fractions. Towards this purpose, we have:

$$\begin{aligned}
& \mathbf{Pr}[A = 1, B = 2, C = 0 \mid class = +] \cdot \mathbf{Pr}[class = +] \\
= \; & \mathbf{Pr}[A = 1 \mid class = +] \cdot \mathbf{Pr}[B = 2 \mid class = +] \cdot \mathbf{Pr}[C = 0 \mid class = +] \cdot \mathbf{Pr}[class = +] \\
\text{(estimate)} \quad = \; & \frac{3}{5} \cdot \frac{2}{5} \cdot \frac{1}{5} \cdot \frac{1}{2} \\
= \; & 6/250
\end{aligned}$$

$$
\begin{aligned}
& \mathbf{Pr}[A=1, B=2, C=0 \mid \mathit{class}=-] \cdot \mathbf{Pr}[\mathit{class}=-] \\
= \; & \mathbf{Pr}[A=1 \mid \mathit{class}=-] \cdot \mathbf{Pr}[B=2 \mid \mathit{class}=-] \cdot \mathbf{Pr}[C=0 \mid \mathit{class}=-] \cdot \mathbf{Pr}[\mathit{class}=-] \\
\text{(estimate)} \quad = \; & \frac{1}{5} \cdot \frac{1}{5} \cdot \frac{3}{5} \cdot \frac{1}{2} \\
= \; & 3/250
\end{aligned}
$$

We therefore conclude that $\mathbf{Pr}[\mathit{class}=+ \mid A=1, B=2, C=0] > \mathbf{Pr}[\mathit{class}=- \mid A=1, B=2, C=0]$.

Problem 4. Suppose that we make an alternative conditional independence assumptions: A and C are independent, conditioned on a class label and a value of B. Decide the larger probability between $\mathbf{Pr}[\mathit{class}=+ \mid A=1, B=2, C=0]$ and $\mathbf{Pr}[\mathit{class}=- \mid A=1, B=2, C=0]$.

Answer. By Bayes Theorem

$$
\begin{aligned}
& \mathbf{Pr}[\mathit{class}=+ \mid A=1, B=2, C=0] \\
= \; & \frac{\mathbf{Pr}[A=1, B=2, C=0 \mid \mathit{class}=+] \cdot \mathbf{Pr}[\mathit{class}=+]}{\mathbf{Pr}[A=1, B=2, C=0]} \\
= \; & \frac{\mathbf{Pr}[A=1, C=0 \mid \mathit{class}=+, B=2] \cdot \mathbf{Pr}[B=2 \mid \mathit{class}=+] \cdot \mathbf{Pr}[\mathit{class}=+]}{\mathbf{Pr}[A=1, B=2, C=0]} \\
= \; & \frac{\mathbf{Pr}[A=1 \mid \mathit{class}=+, B=2] \cdot \mathbf{Pr}[C=0 \mid \mathit{class}=+, B=2] \cdot \mathbf{Pr}[B=2 \mid \mathit{class}=+] \cdot \mathbf{Pr}[\mathit{class}=+]}{\mathbf{Pr}[A=1, B=2, C=0]}
\end{aligned}
$$

where the last step used the given conditional independence assumption. Likewise,

$$
\begin{aligned}
& \mathbf{Pr}[\mathit{class}=- \mid A=1, B=2, C=0] \\
= \; & \frac{\mathbf{Pr}[A=1 \mid \mathit{class}=-, B=2] \cdot \mathbf{Pr}[C=0 \mid \mathit{class}=-, B=2] \cdot \mathbf{Pr}[B=2 \mid \mathit{class}=-] \cdot \mathbf{Pr}[\mathit{class}=-]}{\mathbf{Pr}[A=1, B=2, C=0]}
\end{aligned}
$$

Now we compare the numerators of the two fractions:

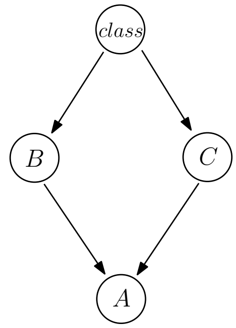
$$
\begin{aligned}
& \mathbf{Pr}[A=1 \mid \mathit{class}=+, B=2] \cdot \mathbf{Pr}[C=0 \mid \mathit{class}=+, B=2] \cdot \\
& \mathbf{Pr}[B=2 \mid \mathit{class}=+] \cdot \mathbf{Pr}[\mathit{class}=+] \\
\text{(estimate)} \quad = \; & \frac{1}{2} \cdot \gamma \cdot \frac{2}{5} \cdot \frac{1}{2} = \frac{\gamma}{10}
\end{aligned}
$$

where $\gamma > 0$ is a very small value introduced to avoid probability 0.

$$
\begin{aligned}
& \mathbf{Pr}[A=1 \mid \mathit{class}=-, B=2] \cdot \mathbf{Pr}[C=0 \mid \mathit{class}=-, B=2] \cdot \\
& \mathbf{Pr}[B=2 \mid \mathit{class}=-] \cdot \mathbf{Pr}[\mathit{class}=-] \\
\text{(estimate)} \quad = \; & \gamma \cdot \gamma \cdot \frac{1}{5} \cdot \frac{1}{2} = \frac{\gamma^2}{10}
\end{aligned}
$$

We thus conclude that $\mathbf{Pr}[\mathit{class}=+ \mid A=1, B=2, C=0] > \mathbf{Pr}[\mathit{class}=- \mid A=1, B=2, C=0]$.

Problem 5*. Based on the following Bayesian network, decide the larger probability between $\mathbf{Pr}[\mathit{class}=+ \mid A=1, B=1, C=0]$ and $\mathbf{Pr}[\mathit{class}=- \mid A=1, B=1, C=0]$.



Answer.

$$
\begin{aligned}
& \mathbf{Pr}[\mathit{class} = + \mid A = 1, B = 1, C = 0] \\
= \; & \frac{\mathbf{Pr}[A = 1, B = 1, C = 0 \mid \mathit{class} = +] \cdot \mathbf{Pr}[\mathit{class} = +]}{\mathbf{Pr}[A = 1, B = 1, C = 0]} \\
= \; & \frac{\mathbf{Pr}[A = 1 \mid B = 1, C = 0] \cdot \mathbf{Pr}[B = 1 \mid \mathit{class} = +] \cdot \mathbf{Pr}[C = 0 \mid \mathit{class} = +] \cdot \mathbf{Pr}[\mathit{class} = +]}{\mathbf{Pr}[A = 1, B = 1, C = 0]}
\end{aligned}
$$

where the last step used the conditional independence assumptions implied by the Bayesian network. Likewise,

$$
\begin{aligned}
& \mathbf{Pr}[\mathit{class} = - \mid A = 1, B = 1, C = 0] \\
= \; & \frac{\mathbf{Pr}[A = 1 \mid B = 1, C = 0] \cdot \mathbf{Pr}[B = 1 \mid \mathit{class} = -] \cdot \mathbf{Pr}[C = 0 \mid \mathit{class} = -] \cdot \mathbf{Pr}[\mathit{class} = -]}{\mathbf{Pr}[A = 1, B = 1, C = 0]}
\end{aligned}
$$

It remains to compare the numerators of the two fractions:

$$
\begin{aligned}
& & & \mathbf{Pr}[A = 1 \mid B = 1, C = 0] \cdot \mathbf{Pr}[B = 1 \mid \mathit{class} = +] \cdot \\
& & & \mathbf{Pr}[C = 0 \mid \mathit{class} = +] \cdot \mathbf{Pr}[\mathit{class} = +] \\
& \text{(estimate)} & = \; & \frac{1}{3} \cdot \frac{2}{5} \cdot \frac{1}{5} \cdot \frac{1}{2} = \frac{1}{75}
\end{aligned}
$$

$$
\begin{aligned}
& & & \mathbf{Pr}[A = 1 \mid B = 1, C = 0] \cdot \mathbf{Pr}[B = 1 \mid \mathit{class} = -] \cdot \\
& & & \mathbf{Pr}[C = 0 \mid \mathit{class} = -] \cdot \mathbf{Pr}[\mathit{class} = -] \\
& \text{(estimate)} & = \; & \frac{1}{3} \cdot \frac{2}{5} \cdot \frac{3}{5} \cdot \frac{1}{2} = \frac{1}{25}
\end{aligned}
$$

We thus conclude that $\mathbf{Pr}[\mathit{class} = + \mid A = 1, B = 1, C = 0] < \mathbf{Pr}[\mathit{class} = - \mid A = 1, B = 1, C = 0]$.