CSCI3160: Special Exercise Set 13

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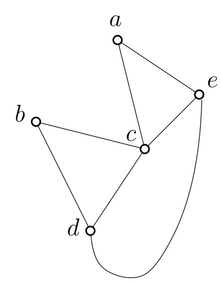
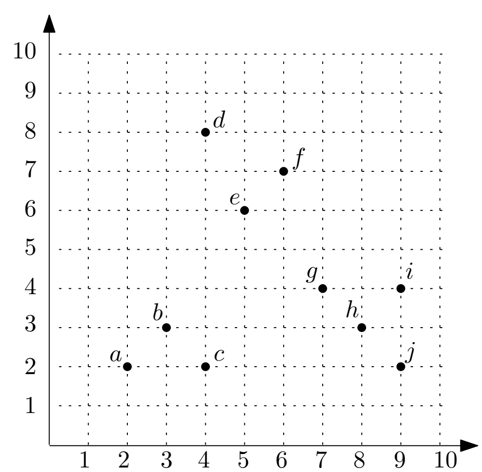
Problem 1. Consider $\mathcal{S} = \{$`arid`, `dash`, `drain`, `heard`, `lost`, `nose`, `shun`, `slate`, `snare`, `thread`$\}$. Given a set L of letters, we call L a *hitting set* if every word in $\mathcal{S}$ uses at least one letter in L. Our goal is to find a hitting set of the smallest size. Re-formulate the problem as a set cover problem.

Problem 2 (2022 Fall Final Exam Problem). Let $G = (V, E)$ be a simple undirected graph. A *5-cycle* is a cycle with 5 edges. We say that a subset $D \subseteq E$ is a *5-cycle destroyer* if removing the edges of D destroys all the 5-cycles in G, namely, $G' = (V, E \setminus D)$ has no 5-cycles. For example, if G is the graph below, there is only one 5-cycle *acbdea*; a 5-cycle destroyer is $\{\{d, e\}\}$, and so is $\{\{d, e\}, \{c, d\}\}$.



Let D^* be a 5-cycle destroyer with the minimum size. Design an algorithm to find a 5-cycle destroyer of size $O(|D^*| \cdot \log |V|)$ in time polynomial to $|V|$.

Problem 3. Consider the following set P of points:



Run the k-center algorithm on P with $k = 3$. Suppose that the first center has been chosen to be f. Show what are the second and third centers found by the algorithm?

Problem 4. The k-center problem we defined in the lecture is on a set P of 2D points. Extend the problem definition to 3D space and design a 2-approximate algorithm.

Problem 5. Explain how the k-center algorithm can be implemented in $O(nk)$ time. You can assume that the Euclidean distance between any two points can be calculated in $O(1)$ time.