CSCI3160: Special Exercise Set 10

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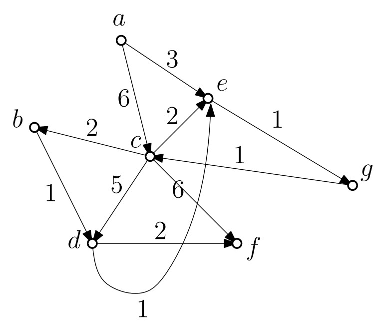
Problem 1. Consider the weighted directed graph below.



Run Dijkstra's algorithm starting from vertex a. Recall that the algorithm relaxes the outgoing edges of every other vertex in turn. Give the order of vertices by which the algorithm relaxes their edges.

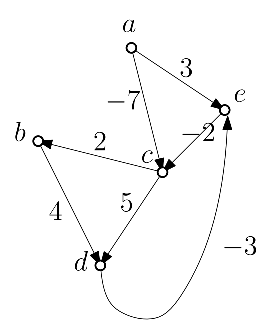
Problem 2. Consider a simple directed graph $G = (V, E)$ where each edge $(u, v) \in E$ carries a non-negative weight $w(u, v)$. Given two vertices $u, v \in V$, function $spdist(u, v)$ represents the shortest path distance from u to v. Given a vertex $v \in V$, denote by $\text{IN}(v)$ the set of in-neighbors of v. Let s and t be two distinct vertices in G. Prove:

$$spdist(s,t) = \min_{v \in IN(t)} \{spdist(s,v) + w(v,t)\}.$$

(Hint: First prove LHS $\leq$ RHS, and then prove $\geq$.)

Problem 3. Give a counterexample to show that Dijkstra's algorithm does not work if edge weights can be negative.

Problem 4. Consider the weighted directed graph $G = (V, E)$ below.



Set the source vertex to a and run Bellman-Ford's algorithm, which performs 4 rounds of edge relaxations. Show the $dist(v)$ value of every $v \in V$ after each round.

Problem 5. The Bellman-Ford algorithm presented in the lecture computes only the shortest-path distance from the source vertex s to every vertex. Extend the algorithm to output a shortest-path tree of s. Your modified algorithm must still terminate in $O(|V||E|)$ time.

Problem 6 (SSSP with Unit Weights). Let us simplify the SSSP problem by requiring that all the edges in the input directed graph $G = (V, E)$ take the *same* positive weight, which we assume to be 1. Give an algorithm that solves the SSSP problem in $O(|V| + |E|)$ time.