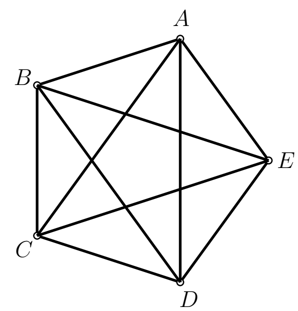
CSCI3160: Quiz 3

Name: Student ID

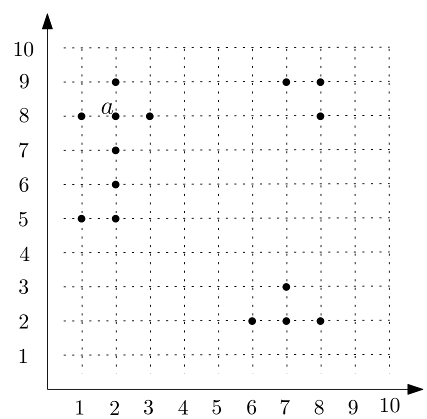
Problem 1 (30%). Let G be the complete graph shown below.



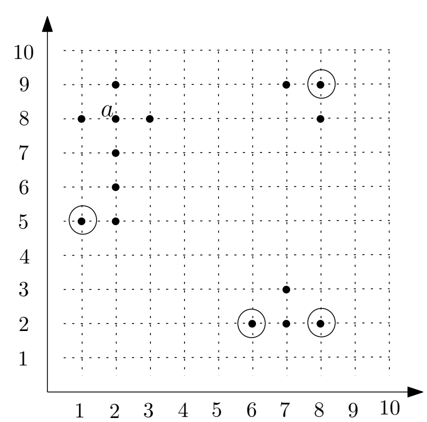
Recall that our 2-approximate TSP (traveling salesman problem) algorithm computes a walk and then generates a Hamiltonian cycle from the walk. If the walk is $ABDEDBCBA$, what is the Hamiltonian cycle returned?

Answer. $ABDECA$.

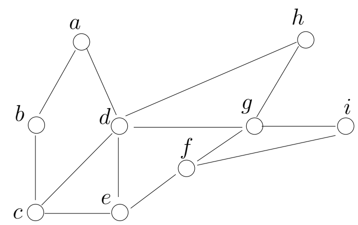
Problem 2 (30%). Consider the set of points shown in the figure below. Suppose that we run the k-center algorithm discussed in class with point a as the first center. Run the algorithm with $k = 5$. Circle the centers returned in the figure.



Answer.



Problem 3 (40%). Let $G = (V, E)$ be an undirected simple graph. A *matching* is a subset $M \subseteq E$ such that no two edges in M share a common vertex. Let OPT be the maximum size of all possible matchings. For example, OPT $= 4$ for the graph below, as is the size of the matching comprising edges $\{a, b\}, \{c, d\}, \{e, f\}, \{g, h\}$.



Consider the algorithm below:

algorithm
$M = \emptyset$
while there is an edge $e \in E$ having no common vertices with the edges in M **do**
    add e to M
return M

Prove: the above algorithm returns a matching with size at least OPT/2.

Answer. Let S be the set of vertices of the edges in M. By how our algorithm runs, we have $|S| = 2|M|$; furthermore, every edge in G must be incident on at least one vertex in S.

Consider any optimal matching M^*. We argue that $|M^*| \leq |S|$. To prove this, for each edge $\{u, v\} \in M^*$:

- if $u \in S$, we ask u to pay a dollar;
- if $v \in S$, we ask v to pay a dollar.

At least one dollar is paid for $\{u, v\}$ because either u, or v, or both are in S. No vertex $u \in S$ is asked to pay twice because M^* can have at most one edge incident on u. Therefore

$$|M| \leq \text{total number of dollars paid} \leq |S|.$$

It now follows that OPT $= |M^*| \leq |S| = 2|M|$.