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Architectures of Mathematics Beliefs: Individual and School-Level Differences Among Hong Kong Primary 6 Students

Ming Ming Chiu
Department of Educational Psychology
The Chinese University of Hong Kong

Ngai-ying Wong, Chi-chung Lam, and Ka-ming Patrick Wong
Department of Curriculum and Instruction
The Chinese University of Hong Kong

Frederick Koon-shing Leung and Ida Ah-chee Mok
Department of Curriculum Studies
The University of Hong Kong

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Correspondence concerning this article should be addressed to Ming Ming Chiu, Department of Educational Psychology, Faculty of Education, The Chinese University of Hong Kong, Shatin, N.T., Hong Kong. E-mail: mingming@cuhk.edu.hk

Students' beliefs influence their learning, so teachers can help students learn by cultivating constructive beliefs and amending harmful ones. Teacher influence is likely greater if student beliefs are loosely structured but weaker if student beliefs are coherent and tightly structured. We examined the structure of students' beliefs by analyzing the questionnaire responses of 2,736 Hong Kong Primary 6 students. Students' mathematics beliefs about formulas, learning by understanding, usefulness, easiness, and interest showed a strong and stable nested structure. This nested structure included a general factor and specific factors for formula, usefulness, easiness and interest. Individual differences accounted for most of the variation among student beliefs (on average, 92% at the student level and 8% at the school level). Several of these beliefs correlated with gender, tutoring, and time spent doing mathematics homework.

Key words: mathematics beliefs; knowledge structure; mathematics education

Student beliefs influence their behavior. Previous research has shown that students' beliefs affect their study effort and their learning strategies (e.g., Fenema & Petereson's [1985] autonomous learning behaviors), and hence, their academic achievement (e.g., McLeod, 1994). For example, students who believe that all mathematics problems can be solved by procedures shown by their teacher are less likely to solve problems that require modifying or combining procedures (Lampert, 1990; Schoenfeld, 1992). Meanwhile, students who believe that mathematics is useful, interesting and important are more likely to work hard (Seegers & Boekaerts, 1993). Moreover, they are more likely to use productive strategies and thereby improve their grades (Pokay & Blumenfeld, 1990). By changing students' harmful beliefs into beneficial ones, a teacher can help students improve their learning behaviors, and ultimately, their learning.

Past studies have not examined the structure of students' beliefs, which affects the degree to which teachers can change students' beliefs. If students' beliefs are tightly linked together, breaking their structure to change their beliefs would be difficult. By contrast, loosely linked beliefs receive little support from related beliefs and hence, are easier to change. Under-

standing the structure of student beliefs helps teachers estimate the difficulty of changing students' beliefs and choose between holistic and piecemeal strategies to improve their students' leaning behaviors.

Past studies have examined the effects of teaching on a few classes of students (e.g. Schoenfeld, 1988), but none have conducted large scale studies to consider the extent to which students' mathematics beliefs originate from societal, classroom, or individual influences. We do this by considering the variation of student beliefs across schools and classrooms. The variation of student beliefs across classrooms reflects the degree of teacher influence. If student beliefs differ substantially across classrooms, then classroom practices likely affect students' beliefs.

This large-scale study extends the research on student beliefs by examining their structures and potential influences on them using advanced statistical analyses. In particular, we examine whether gender, tutoring and schools affect students' beliefs and whether these beliefs affect their study time. We analyzed the questionnaire responses of 2,736 Hong Kong primary 6 students. Hong Kong students are of particular interest because of their top rankings in international mathematics comparisons (e.g., Mullis et al., 2000; OECD, 2003).

Theoretical Framework

A belief is a proposition whose meaning is represented in a person's mental system and treated as if it were true (Gilbert, 1991). A person typically changes his or her beliefs only if they conflict with other ideas and beliefs (Spinoza, 1982). For example, because of their lack of experience, young children tend to accept everything they see uncritically (De Corte, Op't Eynde, & Verschaffel, 2002). By providing a context for perceiving and understanding the world, beliefs play an important emotional and motivational role in learning and problem solving (McLeod, 1994). In this section, we discuss past research on different types of mathematics-related beliefs, the structure of those beliefs and possible influences on them.

Types of Beliefs

Mathematics-focused beliefs. A student's mathematics-related beliefs can be divided into beliefs about mathematics education (*mathematics-focused beliefs*) and beliefs about oneself (*self-focused beliefs*). The former can be further divided into beliefs about properties of mathematics (epistemological beliefs) and beliefs about learning mathematics.

Several mathematics educators have challenged the traditional view of mathematics as a fixed body of absolute facts and procedures dealing with quantities and forms (e.g., Schoenfeld, 1992). Instead, they argue for a view of mathematics as a human activity involving patterns and problem solving. However, studies show that most students share the traditional view (e.g., Schoenfeld). Consider the following list of mathematics beliefs, based on Lampert (1990):

a. Knowing math is recalling and using the correct procedure.
b. Doing mathematics is following the teacher's mathematics procedures.
c. An answer to a mathematics problem is correct if the teacher says it is.
d. Mathematics knowledge is certain.
e. Doing mathematics is giving the correct answer quickly.

Schommer (1990) noted that many of these mathematics beliefs resembled Hofer and Pintrich's (1997) general epistemological categories of simplicity, knowledge sources, justification, and certainty (corresponding to Lampert's [a] – [d] respectively). This correspondence raises the tantalizing possibility that these mathematics beliefs are domain-specific forms of general epistemological dimensions.

Students also have beliefs about learning mathematics that reflect their epistemological beliefs. Consider the following list of learning beliefs (for more extensive lists, see Schoenfeld, 1992).

1. I learn mathematics by memorizing.
2. Learning mathematics is learning procedures.
3. Learning mathematics requires learning when to use each formula.
4. I do not need to learn why a mathematics rule is correct.

5. If I learned the mathematics well, I can do each problem in less than five minutes.

Note how these learning beliefs relate to the previous list of epistemological beliefs. If students believe that knowing mathematics is recalling and using the teacher's correct procedure (beliefs [a] and [b]), then they are likely to believe that they should learn by memorizing procedures and conditions of use given by the teacher (beliefs [1] to [3]). Likewise, if the teacher ultimately decides if a mathematics answer is correct (belief [c]), a student does not need to learn why a mathematics rule is correct (belief [4]). Lastly, if students expect to do mathematics quickly (belief [e]), then speed indicates the quality of their learning (belief [5]). This comparison suggests that students' beliefs about mathematics and about learning mathematics are closely related to one another.

Self-focused beliefs. Pintrich (1989) organized self-focused beliefs into three components: value, expectancy, and affect. Students' beliefs about the value of mathematics consist of the reasons why students engage in learning and doing mathematics. For example, students may learn mathematics because it has intrinsic value or instrumental value (Eccles et al., 1983). Mathematics has intrinsic value for students who enjoy engaging in the task (e.g., "I think math is interesting"). Mathematics has instrumental value or utility for students who view it as useful ("Math will help me get a good job"). On the other hand, expectancy beliefs are students' perceived likelihood of success in a future task ("Math is easy for me"). Pintrich argued that the affect component consists of emotional reactions to the task and task performance. So, these reactions are not beliefs, but consequences of their beliefs.

A student's value and expectancy beliefs are related to one another in the following ways. Students who value an activity are more likely to exert greater effort and hence have higher expectations of success (Seegers & Boekaerts, 1993). Furthermore, past successes are likely to maintain (or increase) perceived activity value and expectations (Pintrich, 1989). Likewise, past failures may lead students to lower their expectations and

devalue the activity to protect their self-esteem from the damage of a likely future failure. So, students' value and expectancy beliefs are likely closely related.

Structure of Beliefs

Researchers have argued that beliefs cluster around situations and have subjective coherence (Bogdan, 1986). Carter and Yackel (1989) noted that although beliefs originate in specific situations, people strive for a coherent belief system organized around central beliefs. However, beliefs are often psychological rather than logical as people tend to form and hold beliefs that serve their own needs and desires, possibly causing biases in perception and judgment (Snow, Corno, & Jackson, 1996, p. 292). If students have undesirable beliefs that an educator wants to change, the structural strength of these beliefs is a key property to consider. Fragmented beliefs are likely easier to change than tightly linked belief systems.

There are five common candidate belief structures including: (a) one factor, (b) separate factors, (c) hierarchical factors, or (d) nested factors. Ryan (1984) proposed a one-dimensional or single factor model. In the single factor model, a student's observed behaviors are different displays of a single well-defined construct. Consider the recorded sprint times of a student in ten 100 meter (100m) runs. The likely single factor underlying these times is the student's 100m sprinting ability. If students' beliefs were sufficiently coherent to fall along one dimension, then changing one belief would be difficult without changing the entire belief system. However, Glenberg and Epstein (1987) showed that Ryan's scale did not explain their questionnaire data well (little explained variance), suggesting that those participants' general beliefs were not one-dimensional.

Schommer (1990) argued that a multi-dimensional model can fit student beliefs better. In the multi-dimensional or separate factors model, the students' beliefs consist of multiple, independent constructs. Consider a student's times in four 200m races and times spent playing computer games during seven days. The two separate factors are likely 200m sprinting abil-

ity and interest in computer games. Using exploratory factor analysis, Schommer (1990) presented evidence that the following four beliefs were relatively independent: (a) Mathematics ability is innate. (b) Mathematics knowledge consists of isolated facts. (c) Mathematics knowledge does not change. (d) Mathematics ideas are learned quickly or not at all. If beliefs are relatively separate, then changing one particular belief is much easier as it is not tightly linked to other beliefs.

Other possibilities are hierarchical factors and nested factors, both of which include both general and specific characteristics. In a hierarchical factors model (Gustafsson & Balke, 1993), a student has several stable beliefs, each of which reflects a general global mathematical belief. For example, consider the times in nine 400m races and nine 500m races. The stable factors are likely 400m and 500m sprinting abilities, and underlying them is a general sprinting speed ability. If beliefs have a tightly organized hierarchical structure, changing them will be very difficult.

In the nested factors model (Gustafsson & Balke, 1993), a student has a general belief independent of specific beliefs. For example, consider the times in six 30m races and six 40 kilometer (km) races that are held outdoors with no turns. Wind speed, sprinting ability, and marathon ability likely affect racing times. Wind speed is a general factor for all races. Meanwhile sprinting ability is specific to the 30m races and marathon ability is specific to the 40km races. Although easier to change than single or hierarchical belief systems, nested beliefs are more difficult to change than separate beliefs.

Factors Affecting Students' Beliefs

Societal, classroom, and individual factors may affect students' beliefs. Societal expectations can affect student beliefs via parental interactions, stereotypes, and national curricula. Parents often raise their children to have society-valued qualities (e.g., filial piety in Chinese societies; Ho, 1986). Likewise, stereotypes can also affect student beliefs. If few women are employed in technical fields requiring mathematical skills, girls are less

likely to view mathematics as useful and are less likely to put effort into learning it (Leder, 1992). Societal values can also permeate national curricula, for example, Hong Kong's civics syllabus (Morris, 1995).

Researchers generally argue that most mathematics beliefs originate during mathematics instruction as much of students' overt mathematics activity occurs in the classroom (Lampert, 1990; Schoenfeld, 1988, 1992). For example, using classroom observations and student responses to questionnaire items, Schoenfeld's (1988) showed that students' beliefs reflect classroom practices. Likewise, educators have successfully changed students' mathematics beliefs (at least in the short-term) by changing their instruction (Higgins, 1997; Verschaffel et al., 1999). Whether other knowledgeable adults outside of school, such as tutors, affect student beliefs remains an open question.

Individual experiences such as past achievement likely influence a person's self-focused beliefs (Pintrich, 1989). Students with greater past achievement are more likely to perceive mathematics tasks as easier and are more likely to be confident about doing them. Although gender differences in student beliefs may reflect societal influences, they can also reflect ability differences (Leder, 1992; Vermeer, 1997). Boys performed better than girls on many (but not all) spatial tasks at all stages of their lives (Halpen & LaMay, 2000). As spatial skills are highly correlated with mathematics skills in geometry (Casey, Nuttall, & Pezaris, 2001), boys score higher than girls on geometry tests, but not on arithmetic, algebra or probability tests (Hanna, 1986). So, girls' beliefs that they have lower ability and less confidence may be rooted in past failures in geometry (Vermeer). However, these beliefs are not supported by their past achievements in other mathematics topics.

The Hong Kong Context

In this section, we describe Hong Kong's school system, classrooms, parents and students. We also note the role of gender differences in Hong Kong's culture.

School system. Next, we consider how the specific context of Hong Kong affects her students by examining her exam system, mathematics curriculum, parent expectations and societal gender expectations. In Hong Kong, education is the primary path to success and financial gain (McLelland, 1991). Students with the highest university entrance exam scores (regardless of gender) enter elite universities and receive the best job opportunities upon graduation. For example, a high school teacher earns a manual worker's lifetime wages in 15 years while a professor earns it within 5 years. As schools and parents support students' efforts to attain high entrance exam scores, this environment likely affects students' beliefs.

Hong Kong has had a free and compulsory 9-year education system since 1979 (Wong, 1993). Students who remain in school after year 9 (about 90% of the age group) take the Hong Kong Certificate of Education Exam after year 11. About 36% of the age group continue schooling and take exams at the end of either year 12 or year 13 to gain university admission. University places were limited to 2%, 8%, and 18% of the relevant age group in 1980, 1990 and 2000 respectively. To compete for the best secondary schools, primary school students took entrance exams at the end of year 6. (After the collection of this data, secondary school entrance exams were abolished in favor of a lottery system within geographic areas.) The mathematics components of these exams cover a wide range of topics and consist of multiple choice questions exclusively.

Hong Kong classroom teaching. To prepare students for these exams, Hong Kong's primary school curriculum emphasized breadth over depth of understanding (Biggs, 1996; Morris, 1995). Furthermore, primary schools' reputations depend in part on how many of their students enroll at the best secondary schools, so teachers often used these difficult, high-stakes exams as models for their class exams (Biggs). Hong Kong textbooks closely follow the examination syllabus. Furthermore, Hong Kong teachers reported that their lessons closely followed both the textbook and the exams (Leung, 1995).

To cover the broad range of topics, teachers often lectured and asked students to apply formulas to solve problems. Teachers encouraged their

students to memorize mathematics facts and formulas to compute precise answers through practice (Biggs, 1991). Hong Kong teacher typically used only one method to solve a problem (in 93% of lessons in Leung, 1995, and in 94% of problems in NCES, 2003). 84% of the lesson problems expected only use of formulas or procedures, while 4% required stating concepts and 13% required making conceptual connections (NCES). Hong Kong teachers did not encourage students to use trial and error and had no lessons involving student investigations or explorations (Leung, 1995). Likewise, the curriculum included many application problems (40% of all problems, NCES) to prepare students for the application problems on the entrance exams (Wong, 1993).

Hong Kong parents and students. Hong Kong parents have high expectations for their children and support their exam preparation (Hau & Salili, 1996). Viewing effort as more important than ability for success, parents encourage their children to put great effort into their studies (Hau & Salili; Lam, Ho, & Wong, 2002). As few Hong Kong parents had any post-secondary education (less than 2%), they typically emphasized traditional study strategies such as practice and memorization (Lam et al.). Many parents also pay for tutoring regardless of their children's achievement (Lam et al.).

Hong Kong students' beliefs reflect these influences. In particular, Hong Kong students want to do well to please both their family and themselves as they believe that their success or failure affects those close to them (Lam et al., 2002). Encouraged by their parents' high expectations, students try to outperform 82% of their age group to enroll in a university. As a result, they have high standards, believe in exerting great effort (Hau & Salili, 1996), have low confidence (Whang & Hancock, 1994), and fear failure (Eaton & Dembo, 1997). Reflecting the Hong Kong curriculum and teaching practices, students view mathematics as memorizing formulas to use them precisely and efficiently (Wong, 2002).

Hong Kong's Confucian culture favors boys over girls, but females are visibly successful both in society and in the classroom. The preferential treatment toward males is visible in many traditional rituals and in the

Confucian view that family members should obey the father in the same way that subjects should obey their king (Confucius, 1997). However, Hong Kong's university entrance exams do not discriminate against girls, and women work in all professional fields, dominating the top government ranks in particular. For example, until recently, a woman, Anson Chan, was the number two ranking official in the Hong Kong government. Moreover, girls outperform boys in all school subjects, engendering a 50% quota of secondary school placements for girls (recently lifted after a successful law-suit against the Hong Kong Education Department).

Study

Past research has shown that students have different types of beliefs that can affect their learning. However, no one has conducted large-scale studies to analyze the structure of these beliefs and the degree to which these beliefs are influenced by factors at different levels (curriculum/societal vs. school vs. individual). This study addresses this research gap by testing the structure of 2,736 students' beliefs, possible influences on them, and their relationships to other variables using advanced statistical analyses. In particular, we examine whether gender, tutoring and schools affected students' beliefs and whether these beliefs affected their study time. As study time is correlated with academic achievement, we use study time as a proxy for academic achievement (Cooper, 2001).

Data Sources

The data that we used was collected through a study commissioned by the Hong Kong Curriculum Development Council (CDC) as part of their review of primary and secondary school mathematics (Wong, Lam, Leung, Mok, & Wong, 1999). We used only a portion of the data collected.

Sample Design & Response Rates

The survey went through a two-step random sampling procedure. First, 90

primary schools were randomly sampled out of all local government and government-subsidized schools (714). Of these schools, 90% agreed to participate. Then, in each chosen primary school, one 6th grade class was selected at random and given a questionnaire to complete. The overall return rate was 95%, ultimately yielding 2,736 student respondents. The response rates of each school differed but showed no obvious geographical or socio-economic pattern.

Procedure for Collecting Data

The primary 6 mathematics teachers administered this questionnaire to their students during mathematics class time, assuring them of their anonymity. The students had up to 30 minutes to fill out the questionnaire in their classroom.

Measures/Variables

The short questionnaire with 27 items was in Chinese, the mother tongue of the students (see English translation in Appendix). We hypothesized that 24 of these items reflected five beliefs (see Appendix for English translations of these items) while the other 3 were likely explanatory or outcome variables. Students responded to belief items on a 5-point Likert scale (unless otherwise noted: 1 =strongly disagree ... 5 = strongly agree).

Mathematics-focused belief items addressed epistemology (formula) and mathematics learning (by understanding). Students answered four questions about the centrality of formulas to mathematics, from a questionnaire created and tested by Lam, Wong, and Wong (LWW; 1999) in the same Hong Kong context. Students also rated the importance of memorizing and understanding to learning mathematics, from a questionnaire created and tested by Wong and Cheng (WC; 1991) in Hong Kong.

Self-focused mathematics belief items addressed task value (usefulness, interest) and expectation (easiness). Students answered five questions drawn from the LWW questionnaire regarding whether mathematics was useful to them. They also rated their degree of interest in mathematics (four ques-

tions from the WC questionnaire). Students rated their perceived easiness of 12 recently taught mathematics topics. The topic names were taken from popular textbooks for primary classes. Students rated the easiness of these topics on a 5-point Likert scale (1 = very difficult ... 5 = very easy).

Students answered three questions regarding gender, tutoring, and homework time. All students indicated their gender. They were classified as tutored if they had a private tutor or attended tutorial classes outside of school. Lastly, they were asked to write down how many hours they spent doing mathematics homework in the last week.

Multiple Imputation of Values for Missing Data

Students did not answer every question, resulting in missing data (1.57% of the total data used in this study). Missing data can lead to the following problems: (a) loss of efficiency, (b) complication in data handling and analysis, and (c) biases due to differences between the observed and unobserved data (Schafer, 1997). To address these issues, we imputed values for the missing data with Markov Chain Monte Carlo multiple imputation (Schafer, 1997). Research studies using computer simulations have shown that other approaches to missing data (pairwise deletion, listwise deletion, mean substitution and simple imputation) do not address the above concerns as effectively (Gold & Bentler, 2000).

Analyses

We used the following procedures to analyze the structure of students' mathematical beliefs, the effect of beliefs on homework time and the potential predictors of students' mathematical beliefs. Unless otherwise specified below, we used LISREL software (Joreskog & Sorbom, 2001) to do the analyses.

Consistency of each belief. First, we tested if student responded consistently to each set of question items. We did so by estimating one-factor congeneric measurement models via a confirmatory factor analysis (CFA) on the item responses' polychoric correlation and asymptotic covariance

matrices (Joreskog & Sorbom, 2001). Unlike parallel and tau-equivalent models, one factor congeneric models allow both the loadings and estimated measurement errors to differ for each variable (Joreskog & Sorbom). Next, we estimated the reliability of each congeneric factor using composite scale reliability coefficients (r_c), which are more precise than Cronbach's alpha (α; Rowe & Rowe, 1997).

Then, we tested the model's goodness of fit. Used Monte Carlo simulation studies, Hu and Bentler (1999) showed that using a combination of the standardized root mean squared residual (SRMSR) and one of the following indices tends to minimize Type I and Type II errors under many conditions. The indices include Tucker-Lewis Index (TLI), incremental fit index (IFI), and root mean squared error of approximation (RMSEA).

(For SRMSR, 0.08 or less indicates a good fit. A value between 0.08 and 0.10 shows a moderate fit. Greater than 0.10 indicates a poor fit. For RMSEA, 0.06 or less shows a good fit. A value between 0.06 and 0.10 indicates a moderate fit, and one greater than 0.10 indicates a poor fit. For TLI and IFI, 0.96 or higher indicates a good fit. Between 0.90 and 0.96 indicates a moderate fit. Less than 0.90 indicates a poor fit.)

Variance of student beliefs at different levels. We tested for influences on students' beliefs by analyzing the variance of student beliefs at different levels. If societal or curricular influences dictated student beliefs, students' responses would be nearly identical, with very little variance. On the other hand, if differences in teaching or classroom experiences primarily affected student beliefs, then most of the variance would be at the school level. Lastly, if individual differences accounted for differences in student beliefs, then most of the variance would be at the student level.

We tested these possibilities by doing multilevel variance component analysis for each factor using *MLn* software (Rasbash & Woodhouse, 1995). (Multi-level analysis is also called hierarchical linear modeling or HLM [Bryk & Raudenbush, 1992].) We also ran multilevel variance component analysis for each variable to test for the robustness of the above result. If the data showed significant school level variance, a multi-level analyses

such as multi-level confirmatory analysis (M-CFA) must be used. Otherwise, a single level CFA would suffice.

Structure of beliefs. We tested the structure of the beliefs against various factor models: (a) single factor, (b) separate factors, (c) hierarchical factors, or (d) nested factors (Gustafsson & Balke, 1993). As mathematics-focused beliefs and self-focused beliefs could have separate sub-structures, we tested the factor structures of the following data sets: (a) mathematics-focused beliefs, (b) self-focused beliefs, and (c) all beliefs. We also tested whether the model of the best sub-structures of (a) and (b) fit the overall data. (See Hu & Bentler, 1999, for a discussion of goodness of fit measures.)

Next, we examined the extent to which each factor explained differences in students' beliefs. We did so by comparing the percentages of explained variance accounted for by each factor (from the best fitting CFA or M-CFA model).

Explanatory model. Using the best factor structure model, we computed composite scores for each belief factor (Joreskog & Sorbom, 2001). These composite scores were single indices of their component terms, each weighted by the factor score to minimize measurement error.

We used these composite scores in a structural equation model (SEM) or a multi-level structural equation model (M-SEM) if needed (Joreskog & Sorbom, 2001), to test our model of how gender, tutoring, beliefs and mathematics homework time affected one another (see Figure 1). Attempts to use the original variables rather than composite scores in an M-SEM failed to converge, which can occur with complex models, so we used composite scores for each belief factor (Joreskog & Sorbom). We removed non-significant links to obtain the final model. For an M-SEM, we kept the links at both school- and student-levels if either one was significant. We computed reduced form squared multiple correlations (RF-SMCs) to estimate the explained variances, also known as the coefficients of determination (Joreskog & Sorbom). We reported standardized coefficients to help readers compare results across analyses.

We also ran additional analyses in which we removed homework time

outliers more than three standard deviations away from the mean. As the results were similar, we reported the results using the entire data set. An alpha level of .05 was used for all statistical tests.

Results and Discussion

The five categories of questions (formula, learning by understanding, usefulness, easiness, interest) all yielded one factor congeneric measure-

Table 1 Stable Congeneric Factors Estimated by Confirmatory Factor Analysis With Two Measures of Reliability

Variable	Factor loading	SE	Uniqueness	RMSEA	SRMSR	TLI	IFI	r_c	α
Formula				.01	.01	1.00	1.00	.768	.705
Formula_1	.49	.02	.76						
Formula_2	.69	.02	.53						
Formula_3	.70	.02	.52						
Formula_4	.73	.02	.46						
Learning by Understanding				.06	.02	.97	.99	.813	.744
Understand_1	.79	.02	.38						
Understand_2	.53	.02	.72						
Understand_3	.65	.02	.58						
Understand_4	.78	.02	.39						
Usefulness				.05	.02	.97	.99	.793	.730
Useful _1	.74	.02	.45						
Useful _2	.78	.02	.40						
Useful _3	.55	.02	.70						
Useful _4	−.59	.02	.65						
Useful _5	−.52	.02	.73						
Easiness				.07	.06	.92	.95	.854	.815
Easiness_1	.62	.02	.62						
Easiness_2	.64	.02	.59						
Easiness_3	.77	.01	.41						
Easiness_4	.66	.02	.56						
Easiness_5	.66	.02	.56						
Easiness_6	.67	.01	.55						
Easiness_7	.64	.02	.58						
Easiness_8	.60	.02	.64						
Interest				.00	.00	1.00	1.00	.760	.719
Interest_1	.69	.02	.53						
Interest_2	.76	.02	.42						
Interest_3	.68	.02	.53						

Note: RMSEA = Root Mean Squared Error of Approximation; SRMSR = Standardized Root Mean Residual; TLI = Tucker-Lewis Index; IFI = Incremental Fit Index; r_c = composite score reliability coefficient; α = Cronbach's α.

ment models with good fits, and with fairly high reliability coefficients (see Table 1). Thus, these beliefs were likely stable across different situations. Several questions did not contribute to these factors (three learning by understanding questions, one interest question, and four easiness questions, see Appendix).

Most students agreed that mathematics consisted of formulas ($M = 3.42$, $SD = 0.75$) and had moderate views on the importance of understanding to learning ($M = 2.96$, $SD = 0.55$). Most students viewed mathematics as useful ($M = 3.93$, $SD = 0.72$) and many mathematics topics as relatively easy ($M = 3.60$; $SD = 0.73$). Lastly, students had varying views on whether mathematics was interesting ($M = 3.08$; $SD = 0.69$).

Variance of Student Beliefs at Different Levels

We considered influences on student beliefs by examining the five stable factors' variances at different levels: (a) overall, (b) at the classroom level, and (c) at the student level. All factors showed substantial variance, with standard deviations ranging from 0.55 to 0.75 (see Table 2). So, neither curriculum nor society dictated students' beliefs as they varied widely among students.

Table 2 Classroom Level % of Total Variance, and Student Level % of Total Variance for Each Belief Factor

Belief factor	School Level %	Student Level %
Formula	9	91
Learning by understanding	6	94
Usefulness	7	93
Easiness	9	91
Interest	11	89
M	8.4	91.6
SD	1.6	1.6

Neither school nor classroom practices primarily determined students' different beliefs. School level variances were low ($M = 8\%$; $SD=2\%$; range = 6% to 11%), showing that students from different classrooms did not show large differences in their beliefs. Regardless of whether classroom

practices were similar or different across schools, they were not the major cause of students' different beliefs.

Most of the variance occurred at the student level (M=92%; SD=2%; range = 89% to 94%), indicating that student differences primarily caused belief differences. Variance components models of student responses to each questionnaire item showed similar results.

Structure of Student Beliefs

The most likely structure for student's mathematics-related beliefs was a nested model (see Table 3). Nested models showed the best fit for mathematics-focused beliefs and self-focused beliefs as well. The preferred nested model for mathematics-focused beliefs consisted of a general factor aligning the beliefs that (a) mathematics is formulas, and (b) learning mathematics does not entail understanding. It also included separate FORMULA and LEARNING BY UNDERSTANDING factors. Likewise, the preferred nested model for self-focused beliefs included a general factor aligning the beliefs that mathematics is (a) useful, (b) easy, and (c) interesting. It also included separate USEFULNESS, EASINESS and INTEREST factors.

For the entire data set, a general ALL factor along with four separate

Table 3 Goodness of Fits for Various M-Cfa Structure Models of Stable Belief Factors, Sorted From Best Fit to Worse Fit Within Each Data Set

Data	Model	RMSEA	χ^2	df	p
Math only	Nested	.01	31	29	.37
	Separate	.04	123	39	.00
	Hierarchical	.04	123	33	.00
	Single	.15	1268	40	.00
Self only	Nested	.04	629	192	.00
	Separate	.05	825	200	.00
	Hierarchical	.05	827	195	.00
	Single	.09	2282	205	.00
All	Nested	.04	1633	476	.00
	Separate	.05	1866	480	.00
	Hierarchical	.05	1981	484	.00
	Math Nested and Self Nested	.07	3978	453	.00
	Single	.08	5408	500	.00

Note: RMSEA = Root mean squared error of approximation.

specific factors showed the best fit. Surprisingly, the ALL factor aligned undesirable mathematics-focused beliefs with desirable self-focused beliefs. Questionnaire items from each factor showed significant loadings for the ALL factor, so it aligned FORMULA, *not* LEARNING BY UNDERSTANDING, USEFULNESS, EASINESS and INTEREST factors along one dimension. (Note that the LEARNING BY UNDERSTANDING items were all negatively worded, so a positive loading indicated *not* LEARNING BY UNDERSTANDING). So, students who viewed that (a) mathematics is formulas and (b) learning it does not entail understanding also viewed mathematics as (c) useful, (d) interesting, and (e) easy. Likewise, many students shared the opposite views of (a) – (e). This showed that many students' mathematics-focused beliefs were inversely related to their self-focused beliefs.

Four factors, FORMULA, USEFULNESS, INTEREST, and EASINESS, showed separate specific effects. There was no LEARNING BY UNDERSTANDING factor, as none of the loadings were significant. Furthermore, INTEREST showed a separate effect at the school level but not at the individual level.

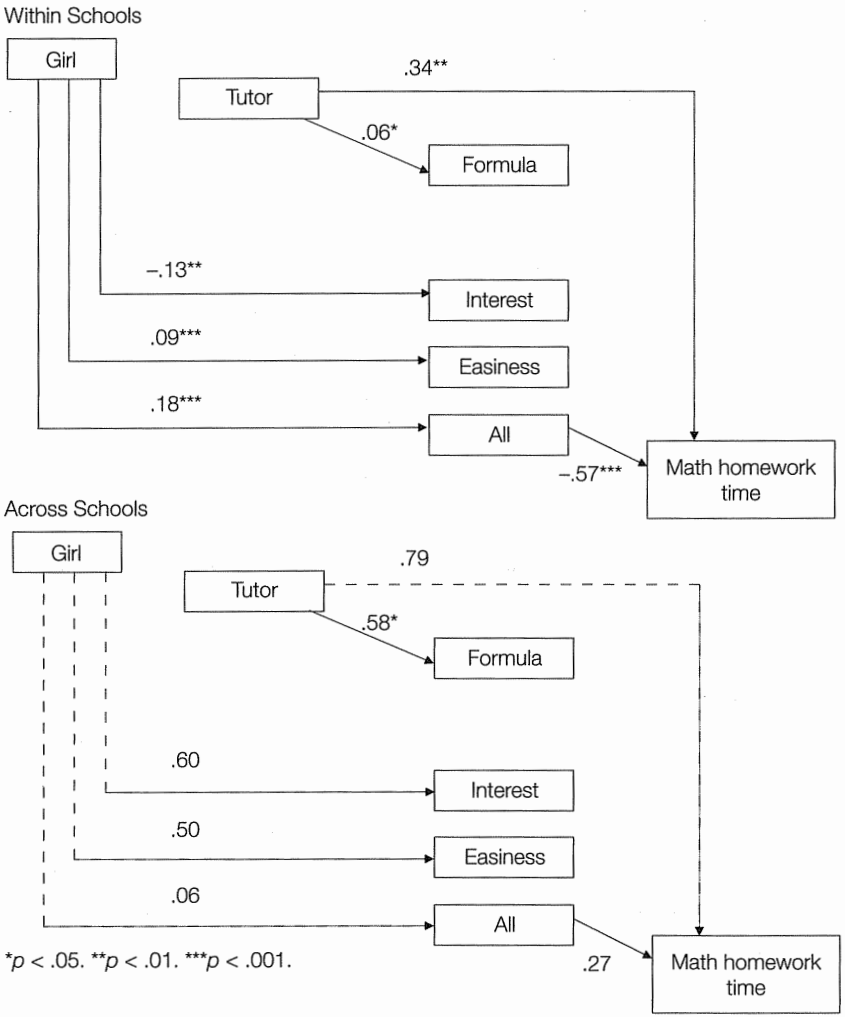
No single factor dominated the explanation of differences in students' beliefs. Students' FORMULA beliefs accounted for the most explained variance, 31% of the explained belief differences, nearly all at the student level. Meanwhile, INTEREST accounted for the least explained variance, 6% (5% at the student level). The general ALL factor accounted for 19% of the explained variance, nearly all at the student level. The EASINESS factor accounted for most of the explained school level differences (5% of the total 7%).

Explanatory Model

Gender and tutoring affected beliefs, which in turn affected time spent doing mathematics homework (Figure 1). As GIRL positively predicted ALL, girls were more likely to view mathematics as formulas, useful, interesting, easy. Likewise, girls were more likely to view learning mathematics as not entailing understanding it. The positive coefficient of GIRL's effect on ALL was larger than the negative coefficient on INTEREST and overall accounted for more of the interest variables than interest did. So, girls tended to be

more interested in mathematics than boys. On the other hand, the positive coefficient of GIRL's effect on EASINESS increased the gender difference further, so girls viewed mathematics as much easier than boys did. Girls also spent less time on mathematics homework as the ALL factor had a negative effect

Figure 1 Multi-level Structural Equation Model of Relationships Between Gender, Tutoring, Mathematics Beliefs and Time Spent Doing Mathematics Homework (RMSEA = .02; χ^2 (24) = 42.8, p =.01; χ^2 / df = 1.78)



on time spent doing mathematics homework. Meanwhile, students with tutors were more likely to view mathematics as formulas and to spend more time doing homework.

All of the above were student-level effects. There was only one significant effect across schools. In schools in which more students received tutoring, students were more likely to view mathematics as formulas. Although this SEM model fits the data well, it explained very little of the variance. The RF-SMCs of mathematics homework time and all beliefs were less than 2%.

Conclusions

The structures of Hong Kong Primary 6 students' mathematics beliefs varied greatly, primarily at the individual level. Students showed complex links among their beliefs, suggesting that changing them would be difficult. The students' stable beliefs correlated with gender, tutoring, and mathematics homework time.

Teachers likely face difficulty in changing the following beliefs: formula, learning by understanding, interest, easiness, and usefulness. Students responded to these sets of questions consistently, indicating stable beliefs. However, the students' responses indicated neither a single belief system (as proposed by Ryan, 1984) nor simple separate beliefs (as Schommer, 1990, argued). Instead, the results showed a general belief factor and separate formula, interest, easiness, and usefulness factors. As this set of beliefs is coherently linked, instruction targeting these beliefs is less likely to be successful than instruction targeting weakly structured beliefs. Of these beliefs, the learning by understanding factor is likely the most difficult to change because it is so strongly linked to the other beliefs that it does have its own separate factor.

The results also suggested that the Hong Kong mathematics curriculum did affect students, but in different ways. The general belief factor aligned undesirable mathematics-focused beliefs and desirable self-focused beliefs. As the mathematics-focused beliefs were consistent with the Hong Kong

mathematics curriculum (Wong, Lam, Wong, Leung, & Mok, 2002), these results suggested that students who viewed school mathematics as useful, interesting and easy were those that accepted the undesirable beliefs supported by the Hong Kong curriculum. In contrast, the Hong Kong curriculum did not support students who viewed mathematics as more than formulas and valued the importance of understanding to learning mathematics. These latter students tended to view mathematics taught in school as useless, uninteresting and difficult. So, Hong Kong's curriculum is producing few students who both enjoy mathematics and try to understand it, and thus, likely few graduates who will create mathematical innovations. This result suggests that if Hong Kong wants to cultivate more mathematically innovative graduates, she must change her curriculum and testing to encourage and reward students who try to understand mathematics.

Innovative curricula or teaching practices are unlikely to change students' beliefs uniformly, as individual differences would likely persist. Classmates received the same instruction from their teacher based on the same curriculum, but they often formed different beliefs. Societal and curricular factors did not successfully dictate any belief to all students. A slight majority of students agreed or strongly agreed that mathematics was useful, likely reflecting the emphasis on applications in the curriculum (Wong et al., 2002). Likewise, a slight majority viewed mathematics as easy or very easy. There was no other student majority on any other belief. So, neither societal nor curricular effects dominated students' beliefs. School and classroom practices accounted for only 10% of the differences among students' beliefs. So, student differences were responsible for most of the variation.

Gender and tutoring also affected students' beliefs. Hong Kong girls viewed mathematics as easier, more useful, and more interesting than boys did, although the effect size was small. These results contrast with those of studies of boys and girls elsewhere (e.g., Vermeer, 1997).

Consistent with Higgins (1997) and Verschaffel et al. (1999), tutor interventions affected students' beliefs, but again the effect size was small.

As the increased belief that mathematics consists of formulas was not a desirable change, educators can not rely on tutors to improve students' beliefs.

Whether these results are generalizable to students from other grades remains an open question. Students of different ages spend different amounts of time engaged in classroom practices, which might lead to stronger or weaker links among beliefs.

Lastly, this study showed how to use statistical techniques to address these issues. We tested the internal validity of the questionnaire data (CFA, r_c) and estimated the structural strength of the beliefs (M-CFA). Using multi-level analyses, we estimated the effects and variances at both the school and student levels. Then, we tested the fit of our model to the data (M-SEM). Together, these methods allow us to answer more questions and to do so with greater precision. Using these methods in future research, we can examine whether these results hold in other countries, identify the origins and consequences of different beliefs, and test the relationship among domain-specific beliefs and general beliefs to target instruction effectively.

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Appendix: Mathematics Beliefs Questionnaire

Questionnaire items are sorted into categories for the reader. Confirmatory factor analysis showed that these items significantly contributed to the category factor and was used in the final models.

Mathematics-focused Beliefs

Epistemological beliefs

Formula:

Formula_1: Mathematics is the manipulation of numbers and symbols.

Formula_2: Mathematics problems are performed by putting numbers into formulas and computing.

Formula_3: Mathematics is the manipulation of formulas.

Formula_4: Mathematics is putting numbers into formulas and finding the answer.

Beliefs about learning mathematics

Importance of Understanding to Learning (all negatively worded):

Understanding_1: We can just learn by reading the formulas in the textbook, without reading the explanations.

Understanding_2: When learning a new topic, I wish that the teacher could tell us the formula right away and not ask us to discover it.

Understanding_3: In learning a new topic, I only care how the formulas are applied in solving problems, not the how the formulas come about.

Understanding_4: Knowing how to calculate suffices in coping with examinations as understanding the content is unimportant.

Self-focused Beliefs

Value

Usefulness:

Useful_1: There are plenty of daily life applications of mathematics.

Useful_2: Mathematics is important as I enter the society in the future.

Useful_3: I think my future career needs mathematics.

Useful_4: I think mathematics does not have much direct applications.

Useful_5: Mathematics is only a mental activity without any applications.

Interest:

Interest_1: I love solving mathematical problems.

Interest_2: My interest in attending mathematics classes is high.

Interest_3: I have interest in mathematical calculations.

Expectations

Easiness:

Easiness_1: Greatest common denominator

Easiness_2: Algebraic problems with equations

Easiness_3: Percentage and its applications

Easiness_4: Circumference

Easiness_5: Square and square root

Easiness_6: Area of circles

Easiness_7: Bearings and location

Easiness_8: Straight line graph