Reasoning of Topological Relations between Imprecise Regions

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Abstract
There are inevitably some errors or uncertainties in spatial data. Such kind of uncertainty will further influence the accuracy of topological relations, which are obtained by reasoning from observation data. In this paper, a determination approach based on relative possibility for topological relations under uncertainty is proposed. First, the effect of positional uncertainty on topological relations is investigated and, statistical modeling of spatial data uncertainty is provided. Then a set of uncertain topological relations for two imprecise regions were built upon a new formal model, proposed by Chen and Deng (2003). Further, some basic functions, which are used for a valid link from positional uncertainty propagating to relation uncertainty, are derived. Finally, a simple example is provided for the illustration of the approach presented in this paper.

I. INTRODUCTION

One of the most fundamental properties of spatial objects in the real world is topological relationship, which has been widely investigated in GIS in recent years (Egenhofer et al, 1991; 1995; Chen et al., 2001; Li et al., 2002). Existing works are based on crisp sets and involved in general topology such as algebraic and point-set topology. However, there are inevitably errors or uncertainties in spatial data, which are used to represent reality world (Goodchild and Gopal, 1989; Guptill and Morrison, 1995; Burrough and Frank, 1996; Shi and Liu, 2000). Further, these errors or uncertainties are propagated with spatial transformations and operations, and will cause the inaccuracy of topological relations obtained by reasoning from observation data (Chen, 1996; Winter, 2000).

In recent years, some scholars like Chen (1996), Clementini (1996), Cohn (1996), Molenaar (1998), and Winter (2000), have paid attention to uncertain topological relations, which are mainly caused by location uncertainty of spatial objects. Several efforts have been made on formal description of topological relations between fuzzy objects. The first one is the Egg-yolk model introduced by Cohn and Gotts (1996), which is built upon region connection calculi (RCC) theory. In the Egg-yolk model, the *egg* is the maximal extent of a vague region and the *yolk* is its minimal extent, while the *white* is the area of indeterminacy. 46 relations are identified based on the so-called limits on the possible 'complete crisping' or precise versions of a vague region. Another is the algebraic model proposed by Clementini and Di Felice (1996). This model is based on the 9-intersection approach. In its definition, a region is composed of a *core* region with a *broad* boundary. The interior and exterior of a region with a broad boundary are open sets, while the broad boundary is a closed set. By the use of 9-intersection approach, 44 different relations are identified. Fundamentally, the above-mentioned two models are the extension of the notation "simple region" into fuzzy simple region. The egg-yolk model, which is based on logic, extends the region into both the egg and its yolk and, both yolk and white are changeable. The approach is similar to which was adopted to analyze topological relations between regions with holes (Egenhofer *et al.*, 1994). The model is difficult to extend it to the relationship between line and region since it is the extension of RCC theory. The algebraic model, which is based on geometric, splits a simple region into yolk and white and the white is 'equivalent' to the crisp boundary of the region. They assume that the extent of the broad boundary of a region is much smaller than of its interior. Actually, this assumption is quasi-topological. It describes an aspect of the object that remains invariant with respect to some common topological transformations such as rotation and scaling. In their assumptions the boundary may be homeomorphic to a one-dimensional circle, and may also be homeomorphic to a two-dimensional region. In addition, Molenaar (1998) developed a determination approach of fuzzy topological relations. In the practical applications it is often more difficult to determine the fuzzy membership, and to interpret the fuzzy membership value for a topological relation. In contrast to fuzzy set approach, Winter (2000) presented a statistical model for quantitative assessment of uncertain topological relations. This model is not suitable for simple line objects, let alone complex objects. As a result, morphological distance used in his paper cannot act as a good bridge between location uncertainty and relation uncertainty.

The remainder of this paper is structured as follows: In Section 2, uncertainty of spatial data (point line, and region) is analyzed and modeled based on the assumption that positional uncertainty of point complies with a 2-dimensional nor-

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mal distribution. By means of a new formal model, a set of topological relations between two imprecise regions is given in Section 3. A combination of statistical modeling of positional uncertainty with description of topological relations is considered, and a determination approach based on relative possibility is proposed in Section 4. Section 5 provides a simple example for illustration of the approach. This paper ends with some discussions and conclusions in Section 6.

II. REPRESENTATION OF UNCERTAIN SPATIAL DATA

Measures of spatial graphic structure under uncertainty

The present paper exclusively focuses on spatial entities with definite boundary locations in the real world, like highway, buildings. In this case, location uncertainty is mainly from digitalization, scanning or field measurement. It is therefore reasonable to assume that positional uncertainty of spatial data is characterized by randomness. Henceforth, the spatial objects with randomness are called imprecise objects, including imprecise point, imprecise line, and imprecise region.

Definition 1: For any imprecise line, L_i, all of its vertex should satisfy $Deg(P_i) \geq 1$, in which $Deg(P_i)$ denotes the connective degree related with vertex P_i. If there is the equation, $Deg(P_i)=1$, the vertex P_i is the boundary point of the imprecise line.

Definition 2: If there is a chain between two imprecise nodes, we call the two imprecise nodes connective. Furthermore, if all pairs of nodes in a planar graph are connective, the graph is also connective.

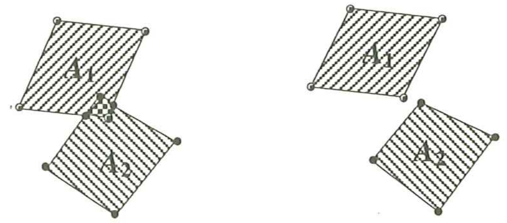
Any graph in the plane, G, is composed of imprecise nodes, imprecise edges and imprecise faces, and the numbers of these elements satisfy the following formula

$$f + n - e = c + 1 \quad (1)$$

where f, n, e are the numbers of face, node and edge, and c is the number of connective branches of G. If a planar graph is connective, i.e. c=1, formula (1) is reduced to

$$f + n - e = 2 \quad (2)$$

Formula (2) is the famous Eula theorem, which is often used to check topological inconsistency. Here it is applied to analyze changes of spatial graphic structure under the effect of uncertainty. Let us look at Figure 1. In case (a), it satisfies the expression $f + n - e = 2$, but not for case (b). In topological aspect, a qualitative change occurs in between them. The topological relations in (a) and (b) are described as '*overlap*' and '*disjoint*', respectively. In the real world the true relation between A_1 and A_2 is possible '*meet*'. Hence, it is necessary to measure the uncertainty for a determination to be some topological relation.



(a) *overlap* (b) *disjoint*

Figure 1. Effects of positional uncertainty on spatial graphic structure

Modeling of spatial data uncertainty

For uncertainty of positional data, there are considerable research documents (Shi, 1994; Gong et al., 1995; Leung and Yan, 1998; Shi and Liu, 2000). Upon the assumption that spatial data uncertainty is characterized by randomness, it can be further modeled statistically. In GIS databases, a point in a 2-dimensional space, P_i, is measured by its coordinate (x, y), and a line or arc L_i represented by some connective points, P_1, P_2, …, P_n, while an area, A, demarcated by a sequence of boundary lines, denoted as $L_1, L_2, \ldots, L_n$. Moreover, positional uncertainty of a point in a line (or region boundary) will further affect location exactness of the line or the region boundary. Geometrically, any point in a line can be linearly represented by the two adjacent end-points (Shi, 1994)

$$P_t = t\,P_i + (1-t)\,P_{i+1} \quad (3)$$

where t is a parameter of splitting ratio of length. Its value equals $|P_i P_t|/|P_i P_{i+1}|$. For simplicity, it is assumed that positional uncertainty of a point complies with a normal distribution. Mathematically, such point may be regarded as a random variable. Moreover, if a random variable can be expressed as a linear combination of other two random variables complying with normal distribution, then the random variable obtained by combination also satisfies a normal distribution. As a result, any interpolation point in line or region boundary, which is determined by formula (3), will be regarded as a random variable complying with a normal distribution. On the basis of this, a random line can be regarded as a normal random process (Shi and Liu, 2000), and a random region as a normal random field (Liu et al., 1998). The geometric projections of their spatial distributions in a 2-dimensional plane correspond to a band-shape region and a donut-shape region (see Figure 2), which are called '*g*-band' and '*g*-donut' in the previous papers, respectively.

III. TOPOLOGICAL RELATIONS BETWEEN IMPRECISE REGIONS

Topological relations and the conceptual neighborhood graph

Currently, several formal models for description of topological relations are available, such as RCC model, 4/9-intersection model, and so on. Clementini et al. (1995) made a comprehensive analysis for these models from the view of identification and representation of topological relations. It is pointed out that the 4-intersection model is identical with the 9-intersec-

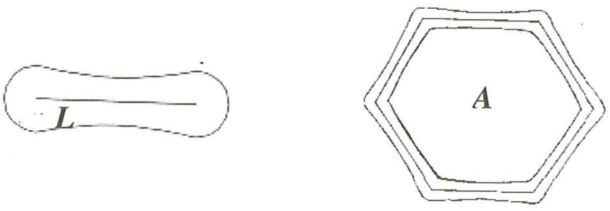


Figure 2. Visual g-band and g-donut of imprecise line and region in a 2-dimensional plane

tion model for identification of topological relations between two simple regions. In addition, the problems for the 4/9-intersection models have been discussed in some research documents, such as Chen et al. (2001) and Li et al. (2000, 2002). The model proposed by Chen et al. (2001), which is called Voronoi-based 9-intersection model, can overcome partial shortcomings of the 9-intersection model, and further the model proposed by Li et al (2002) can distinguish more topological relations than the Voronoi-based 9-intersection model. In recent, Chen and Deng (2003) presented a new 4-intersection-difference model (abbreviated as 4ID model), which consists of two intersection sets and two difference sets, i.e.

$$\gamma(A_1, A_2) = \begin{bmatrix} A_1^{\,o} \cap A_2^{\,o} & A_1 - A_2 \\ A_2 - A_1 & \partial A_1 \cap \partial A_2 \end{bmatrix} \qquad (4)$$

where $A_i^{\,o}$ and ∂A_i (i=1,2) are interior and boundary of A_i, respectively. 4ID model is a binary formal model for topological relation, element of which takes value ϕ or $\neg\phi$. By using of the 4ID model, topological relations between two certain regions in a 2-dimensional space can be distinguished into 8 families. These relations are named '*disjoint*', '*meet*', '*overlap*', '*contains*', '*covers*', '*inside*', '*coveredby*', and '*equal*', as listed in Figure 3. It has been pointed out that topological relations may convert from one caregory to another with changes in geometry of one or two of the involved objects. That is to say that a qualitative change occurs if geometric change of an object affects its topological relationship with respect to another object. Therefore, all of the eight relations in Figure 3 can occur with a certain degree of changes in geometry, including object location, orientation, shape, and size. In order to represent their occurrence rule, a concept of topological distance is presented here.

Let $\delta(*)$ be a function of mapping the values of empty and non-empty onto the integers 0 and 1, defined as:

$$\delta(*) = \begin{cases} 1, & * \neq \phi \\ 0, & * = \phi \end{cases} \qquad (5)$$

Furthermore, topological distance between two relations can be defined as:

$$d_T(\gamma_1, \gamma_2) = \sum_{i=1}^{4} | \delta(s_i^1) - \delta(s_i^2) | \qquad (6)$$

where $d_T(\gamma_1, \gamma_2)$ represents the topological distance between relation γ_1 and relation γ_2 ; s_i^1 and s_i^2 $(1 \le i \le 4)$ are coresponding elements in γ_1 and γ_2, respectively. By using of formula (6), we can calculate the topological distance between any two relations shown in Figure 3. For instance, the topological relations '*disjoint*' and '*overlap*' are described by using the 4ID model as follows:

$$disjoint = \begin{bmatrix} \phi & \neg\phi \\ \neg\phi & \phi \end{bmatrix}, overlap = \begin{bmatrix} \neg\phi & \neg\phi \\ \neg\phi & \neg\phi \end{bmatrix}$$

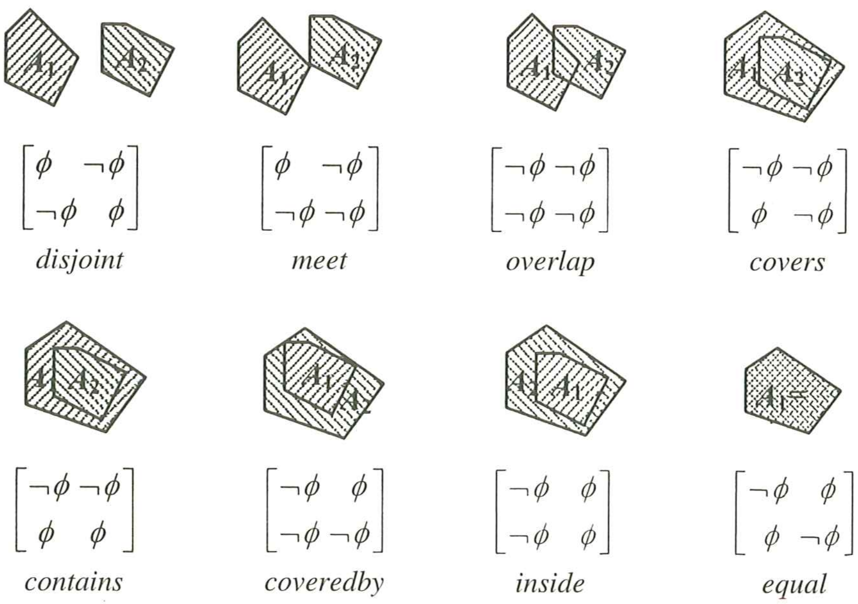


Figure 3. Eight region/region relations based on 4ID model

and then the topological distance is calculated as:

$d_T(disjoint, overlap) = |0-1|+|1-1|+|1-1|+|0-1| = 2$

Similarly, all the distance for other relations may be obtained, as listed in Table 1. It can be seen from Table 1 that the topological distance satisfies the following four properties:

1) $d_T(\gamma_1, \gamma_2) = d_T(\gamma_2, \gamma_1)$;

2) $0 \le d_T(\gamma_1, \gamma_2) \le 4$;

3) $d_T(\gamma_1, \gamma_2) = 0$, If and only if $\gamma_1 = \gamma_2$;

4) $d_T(\gamma_1, \gamma_2) + d_T(\gamma_2, \gamma_3) \ge d_T(\gamma_1, \gamma_3)$

.

It is also shown that the minimal distance between distinct topological relations are 1. In this paper, such two relations with topological distance equal to 1, is defined as neighborhood relations. That means if there satisfies the condition

$$d_T(\gamma_1, \gamma_2) = 1 \qquad (7)$$

γ_1 and γ_2 are then termed of neighborhood relations. Further, we link all pairs of neighborhood relations, thus the conceptual neighborhood graph is set up, as shown in Figure 4. This is particulatly useful to predict what is the most likely relations after a change in geometry of one or two objects occurs.

Under uncertainty circumstance, spatial data uncertainty, in essence, can be regarded as a small deformation of object boundary in location, size, shape, etc. It only possibly changes topological relation from one kind to another, and does not increase or decrease topological categories. As a result, for two imprecise regions, separable topological relations are still 8 families with the same names. In the following, 4ID model is taken as a basis of reasoning topological relations under uncertainty.

Reasoning of topological Relations from observations

In practical application, the topological relation by reasoning

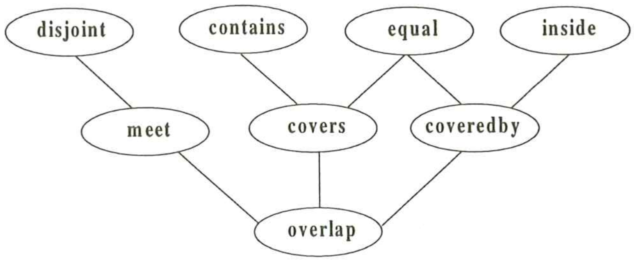


Figure 4. The conceptual neighborhood graph derived by topological distance

from observations may be inconsistent with the true relation. As shown in Figure 1, topological relations between A_1 and A_2 may be described as '*overlap*', or '*disjoint*' with a very small perturbation on one or both of objects, where the true relation is '*meet*'. Sometimes, spatial data uncertainty does not have any effect on description of topological relation. One example is that two *disjoint* objects are very far, and in this case uncertainty may be ignored. Therefore, it is necessary to make further investigation and set up a general model for topological relations under location uncertainty.

The *g*-donut model is utilized to represent an imprecise region. And here we define the region which is demarcated by the exterior boundary of *g*-donut as exterior region, represented by g^+, the region demarcated by the interior boundary of *g*-donut as interior region, represented by g^-. Based on the 4ID model, four relations are defined as follows:

I) $$\gamma(g_1^+, g_2^+) = \begin{bmatrix} (g_1^+)^o \cap (g_2^+)^o & g_1^+ - g_2^+ \\ g_2^+ - g_1^+ & \partial(g_1^+) \cap \partial(g_2^+) \end{bmatrix}$$

II) $$\gamma(g_1^-, g_2^+) = \begin{bmatrix} (g_1^-)^o \cap (g_2^+)^o & g_1^- - g_2^+ \\ g_2^+ - g_1^- & \partial(g_1^-) \cap \partial(g_2^+) \end{bmatrix}$$

Table 1. The topological distance between the eight binary relations

$d_T(-,-)$	*disjoint*	*meet*	*overlap*	*covers*	*contains*	*coveredby*	*inside*	*equal*
disjoint	0	1	2	3	2	3	2	4
meet	1	0	1	2	3	2	3	3
overlap	2	1	0	1	2	1	2	2
covers	3	2	1	0	1	2	3	1
contains	2	3	2	1	0	3	2	2
coveredby	3	2	1	2	3	0	1	1
inside	2	3	2	3	2	1	0	2

III) $\gamma(g_1^+, g_2^-) = \begin{bmatrix} (g_1^+)^o \cap (g_2^-)^o & g_1^+ - g_2^- \\ g_2^- - g_1^+ & \partial(g_1^+) \cap \partial(g_2^-) \end{bmatrix}$

IV) $\gamma(g_1^-, g_2^-) = \begin{bmatrix} (g_1^-)^o \cap (g_2^-)^o & g_1^- - g_2^- \\ g_2^- - g_1^- & \partial(g_1^-) \cap \partial(g_2^-) \end{bmatrix}$

where g_i^+ and g_i^- (i=1, 2) are outer and inner region of *g*-donut of regions A_i. $(g_i^+)^o$ and $\partial(g_i^+)$ are the interior and the boundary of g_i^+, respectively. Similarly, $(g_i^-)^o$ and $\partial(g_i^-)$ denote the interior and the boundary of g_i^-. By the definitions of I)~IV), all of the possible topological relations between A_1 and A_2 can be expressed as:

$$\tilde{\gamma}(A_1, A_2) = <\gamma(g_1^+, g_2^+), \gamma(g_1^+, g_2^-)> \cup <\gamma(g_1^-, g_2^+), \gamma(g_1^-, g_2^-)> \cup <\gamma(g_1^+, g_2^+), \gamma(g_1^-, g_2^+)> \cup <\gamma(g_1^+, g_2^-), \gamma(g_1^-, g_2^-)> \cup \gamma(A_1, A_2) \quad (8)$$

In formula (8), $\gamma(A_1, A_2)$ is the observed topological relation; symbol $<>$ all possible relations. For instance, $<\gamma(g_1^+, g_2^+), \gamma(g_1^+, g_2^-)>$ includes the relations $\gamma(g_1^+, g_2^+)$, $\gamma(g_1^+, g_2^-)$, and some relation (γ_s) that must occurs from $\gamma(g_1^+, g_2^+)$ to $\gamma(g_1^+, g_2^-)$. The relationcan γ_s be determined according to the conceptual neighborhood graph presented in Section 3.1. A simple example is that, the relation γ_s will be '*meet*' if having $\gamma(g_1^+, g_2^+)$ = '*overlap*' and $\gamma(g_1^+, g_2^-)$ ='*disjoint*'. Further, $<\gamma(g_1^+, g_2^+), \gamma(g_1^+, g_2^-)> = \{$'*overlap*', '*meet*', '*disjoint*'$\}$ Apparently, $\tilde{\gamma}(A_1, A_2)$ is likely to include more than one relation. In spatial analysis, we have to make a decision among all possible relations. It thereof is necessary to set up a criterion for such a decision. This issue will be further investigated in the sue section.

IV. A DETERMINATION APPROACH BASED ON RELATIVE POSSIBILITY

Uncertainty in 4ID model

Here, take Figure 1 as an example. By the definition of (8), all possible topological relations between A_1 and A_2 are '*overlap*', '*meet*' and '*disjoint*'. Their corresponding 4IDs are described as follows:

$$\begin{bmatrix} \neg\phi & \neg\phi \\ \neg\phi & \neg\phi \end{bmatrix}, \begin{bmatrix} \phi & \neg\phi \\ \neg\phi & \neg\phi \end{bmatrix}, \begin{bmatrix} \phi & \neg\phi \\ \neg\phi & \phi \end{bmatrix}$$

According to the conceptual neighborhood graph in Figure 4, their order of occurrence under a continuous deformation is: '*overlap*' → '*meet*' → '*disjoint*', or '*disjoint*' → '*meet*' → 'overlap'. Moreover, only an element varies as each of topological changes occurs. For instance, only the element in left-top corner of 4IDs is changed from '*overlap*' to '*meet*'. Thus, the effect of location uncertainty on topological relations is embodied by change of content of one or some elements in 4ID. Furthermore, a determination of '*overlap*' and '*meet*' is to compare the possibility that the element in left-top corner takes $\neg\phi$ and ϕ. It is similar to analyze the change between '*meet*' and '*disjoint*'.

Construction of basic possibility functions

For two imprecise regions, A_1 and A_2, their relation uncertainty can be reduced to uncertainty of relations between points in A_1 and A_2, or uncertainty of relations between points in A_2 and A_1. Its basis is to determine uncertain relations between an imprecise point and an imprecise region. There the determination mainly involves the relations between imprecise points, see Figure 5. Topologically, there are two possible relations, namely, '*equal*' and '*disjoint*' (or 'unequal'). As a result, it needs a quantitative determination of the relations between two imprecise points, as is built upon some basic functions. In the following we will first set up a function for determination of relations between a certain point and an imprecise point.

On the basis of Section 2.2, a further assumption is made that positional uncertainty of a point in 2-dimensional plane complies with a circle normal distribution, i.e. $\sigma_x = \sigma_y = \sigma, \sigma_{xy} = 0$, and the probability density function is as follows:

$$f(x, y) = \frac{1}{2\pi\sigma^2} \exp[-\frac{(x-x_P)^2 + (y-y_P)^2}{2\sigma^2}] \quad (9)$$

where parameter σ is the standard error of coordinate components of $P(x_P, y_P)$. Likewise, an interpolated point obtained by formula (3) also complies with a normal distribution, which is similar to formula (9). Furthermore, the probability of any point falling in the equal density error circle can be calculated by the following expression,

$$\begin{aligned} p((x, y) \subset C_r) &= \iint_{C_r} \frac{1}{2\pi\sigma^2} \exp[-\frac{(x-x_P)^2 + (y-y_P)^2}{2\sigma^2}] dxdy \\ &= \int_0^r \frac{\rho}{\sigma^2} \exp(-\frac{\rho}{2\sigma^2}) d\rho \\ &= 1 - \exp(-\frac{r^2}{2\sigma^2}) \end{aligned} \quad (10)$$

where C_r: $(x - x_P)^2 + (y - y_P)^2 \le r^2$, r the radiu of error circle. It is apparent that the following function

$$F(r) = \begin{cases} 1 - \exp(-\frac{r^2}{2\sigma^2}), & r \ge 0 \\ 0, & r < 0 \end{cases} \quad (11)$$

is a distribution function, specifically the Rayleigh distribution. Considering that the standard error of point (σ_P) equals:

$$\sigma_P = \sqrt{(\sigma_x^2 + \sigma_y^2)} = \sqrt{2}\sigma$$

Therefore formula (10) is simplified into

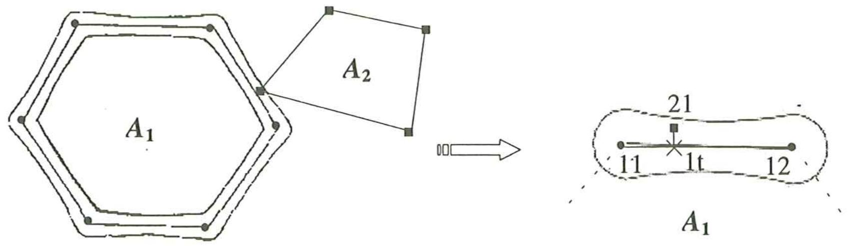


Figure 5. An illustration of determining topological relations for two imprecise regions

$$F(r)=\begin{cases}1-\exp(-\frac{r^2}{\sigma_P{}^2}), & r\ge 0\\ 0, & r<0\end{cases} \quad (12)$$

While for a certain point (a point without error) and an imprecise point, we here presume a normal distribution with its center being the certain point, and standard error being that of the imprecise point. Further, a quantitative determination function for their uncertain relation may be defined as:

$$\begin{aligned}\theta(\gamma='equal')&=1-\lim_{r\to d-}F(r)\\&=1-(1-\exp(-\tfrac{d^2}{\sigma_P^2}))\\&=\exp(-\tfrac{d^2}{\sigma_P^2})\end{aligned} \quad (13)$$

where d is the distance between imprecise point and certain point. In terms of formula (13), if the distance between two points equals zero, then one can make the decision that the relations between two points is '*equal*' with the possibility 1.

If the distance is large enough (larger than $3\sigma_P$), then the possibility of '*equal*' is nearly 0.

For two imprecise points, $P_1(x_1,y_1)$ and $P_2(x_2,y_2)$, their quantitative determination function can be defined as an extension of formula (13), i.e.

$$\theta(\gamma='equal')=\theta_1(\gamma='equal')\times\theta_2(\gamma='equal') \quad (14)$$

Here,

$\theta_1(\gamma='equal')=\exp(-\frac{d^2}{\sigma_1^2})$, and

$\theta_2(\gamma='equal')=\exp(-\frac{d^2}{\sigma_2^2})$

where σ_1 and σ_2 are the standard error of $P_1(x_1,y_1)$ and $P_2(x_2,y_2)$, respectively.

Above definitions are also suitable for the relation between a point and a line under uncertainty. The difference is that another point used is a projection point or one of endpoints in the line, and that d is the shortest distance from the point to the line. For an imprecise point and an imprecise region, there are still three basic relations, namely, '*a point inside a region*', '*a point on the boundary of a region*', and '*a point outside a region*'. We here abbreviate them as '*inside*', '*on*' and '*outside*'. These relations can be determined by using some classic algorithms in computational geometry. It is easily found that there exists such an occurrence order as '*inside*' → '*on*' → '*outside*', or '*outside*' → '*on*' → '*inside*' when the locations of point or/and region change continuously. Therefore, a similar determination function to formula (14) for relation '*on*' can be defined, i.e.

$$\theta(\gamma='on')=\theta(\gamma='equal') \quad (15)$$

and,

$$\theta(\gamma\neq'on')=1-\theta(\gamma='equal') \quad (16)$$

If the observed relation is '*outside*', then we can define the following functions:

$$\theta(\gamma='outside')=\theta(\gamma\neq'on') \quad (17)$$

$$\theta(\gamma='inside')=\exp(-\tfrac{(d+\sigma_1)^2}{\sigma_1^2})\times\exp(-\tfrac{(d+\sigma_2)^2}{\sigma_2^2}) \quad (18)$$

Similarly, if the observed relation is '*inside*', then formulae (17) and (18) become

$$\theta(\gamma='inside')=\theta(\gamma\neq'on') \quad (19)$$

$$\theta(\gamma='outside')=\exp(-\tfrac{(d+\sigma_1)^2}{\sigma_1^2})\times\exp(-\tfrac{(d+\sigma_2)^2}{\sigma_2^2}) \quad (20)$$

where d is the distance from the imprecise point to the imprecise region boundary; the meanings of σ_1 and σ_2 are the standard errors of two imprecise points used for comparisons, respectively.

A determination indicator: relative possibility

For two imprecise regions, A_1 and A_2, let their observated relation be γ_i, which is obtained by reasoning spatial data stored in GIS databases. In terms of basic principle of perturbation, we may deem that the true relation between A_1 and A_2, γ, is one of uncertain relation set $\tilde{\gamma}(A_1,A_2)$. Here, assuming that $\tilde{\gamma}(A_1,A_2)$ obtained by formula (8) includes n relations, denoted by $\gamma_1,\gamma_2,\ldots,\gamma_n$, and that their relative possibility is $\varphi_1,\varphi_2,\ldots,\varphi_n$, respectively. Thus the true topological relations between A_1 and A_2 can be expressed as:

$$\gamma=\varphi_1/\gamma_1+\varphi_2/\gamma_2+\cdots+\varphi_n/\gamma_n \quad (21)$$

Here, '+' denotes a connection symbol, not the sum operation; $\varphi_j=\varphi_{ij}(\gamma=\gamma_j\mid\gamma_i)$ represents the possibility of $\gamma=\gamma_j$ under the observed relation γ_j. It is called the relative possibility in this paper. If having $\varphi_q=\max(\varphi_1,\varphi_2,\cdots,\varphi_n)$, one can determine γ_q to be the true relation between A_1 and A_2. A simple new approach is presented below for comparisons of φ_i $(1 \le i \le n)$.

Let $\gamma_i = \begin{bmatrix} \alpha_{11}^i & \alpha_{12}^i \\ \alpha_{21}^i & \alpha_{22}^i \end{bmatrix}, \gamma_j = \begin{bmatrix} \alpha_{11}^j & \alpha_{12}^j \\ \alpha_{21}^j & \alpha_{22}^j \end{bmatrix}$, let γ_i be the observed relation, and then we define

$$\varphi_{ij}(\gamma = \gamma_j \mid \gamma_i) = \frac{\prod_{k,m=1}^{k,m=2} \theta(\alpha_{km}^j)}{\prod_{k,m=1}^{k,m=2} \theta(\alpha_{km}^i)} \tag{22}$$

where $\theta(\alpha_{km}^i)$, $\theta(\alpha_{km}^j)$ $(1 \le k, m \le 2)$ is respectively the possibility that element α_{km} in γ_i and γ_j takes values α_{km}^i, α_{km}^j. They are calculated by using basic possibility functions mentioned above. At the same time, it is easily found that relative possibility, φ_{ij}, satisfies the following properties:

1) $0 < \varphi_{ij} < +\infty$;
2) $\varphi_{ij} = (\varphi_{ji})^{-1}$;
3) For $j = i$, $\varphi_{ij} = 1$.

V. EXAMPLE

In Figure 6, there are two imprecise region objects, A_1 and A_2, which are from different data layers. Their location and accuracy data are listed in Table 2. The observed relation between A_1 and A_2 is '*contains*'.

At first, the method presented in Liu et al. (1998), is used for the generation of g-donut of A_1 and A_2, denoted by g_1, g_2. In the light of I)~IV) defined in Section 3.2, four relations between them are respectively computed and, on the basis of the 4ID model, they are described as follows,

$\gamma(g_1^+, g_2^+)$ = '*contains*', $\gamma(g_1^+, g_2^-)$ = '*contains*', $\gamma(g_1^-, g_2^+)$ = '*overlap*', $\gamma(g_1^-, g_2^-)$ = '*contains*'.

Furthermore, having

$<\gamma(g_1^+, g_2^+), \gamma(g_1^+, g_2^-)>$ = {'*contains*'};

$<\gamma(g_1^-, g_2^+), \gamma(g_1^-, g_2^-)>$ = {'*contains*', '*covers*', '*overlap*'};

$<\gamma(g_1^+, g_2^+), \gamma(g_1^-, g_2^+)>$ = {'*contains*', '*covers*', '*overlap*'};

$<\gamma(g_1^+, g_2^-), \gamma(g_1^-, g_2^-)>$ = {'*contains*'}.

By the definition of (8), all of possible topological relations between A_1 and A_2, $\tilde{\gamma}(A_1, A_2)$, are expressed as

$\tilde{\gamma}(A_1, A_2)$ = {'*contains*', '*covers*', '*overlap*'}

Now it needs to calculate their relative possibility under the observed relation '*contains*', including:

i) $\varphi(\gamma = \text{'contains'} \mid \text{'contains'})$;

ii) $\varphi(\gamma = \text{'covers'} \mid \text{'contains'})$;

iii) $\varphi(\gamma = \text{'overlap'} \mid \text{'contains'})$.

By the definition of (18), it is obvious that for i) there has

$\varphi(\gamma = \text{'contains'} \mid \text{'contains'}) = 1$

For ii), there only exists a different element in the 4IDs of both '*covers*' and '*contains*'. That is, $\partial A_1 \cap \partial A_2$ takes value ϕ for '*contains*', while $\neg\phi$ for '*covers*'. Therefore, formula (22) is reduced to

$$\varphi(\gamma = \text{'covers'} \mid \text{'contains'}) = \frac{\theta(\partial A_1 \cap \partial A_2 = \neg\phi)}{\theta(\partial A_1 \cap \partial A_2 = \phi)}$$

While in Figure 6, only the point '*21*' in A_2 possibly locates on the boundary of A_1. The possibilities will be calculated below using formulae (15)~(20), i.e.

$$\theta(\partial A_1 \cap \partial A_2 = \neg\phi) = \exp(-\tfrac{d^2}{\sigma_1^2}) \times \exp(-\tfrac{d^2}{\sigma_2^2}) = 0.762$$

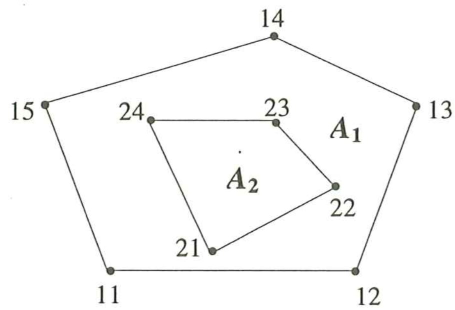


Figure 6. A schematic graph of topological relation between A_1 and A_2

Table 2. Location and accuracy of A_1 and A_2

Point No.	x/m	y/m	σ_x/m	σ_y/m	ρ_{xy}
11	1468.32	8798.56	15.00	15.00	0
12	2383.82	8798.56	15.00	15.00	0
13	2688.97	9513.24	15.00	15.00	0
14	2162.16	9704.58	15.00	15.00	0
15	1252.76	9601.43	15.00	15.00	0
21	1834.52	8805.14	15.00	15.00	0
22	2264.35	9065.84	15.00	15.00	0
23	2170.41	9440.18	15.00	15.00	0
24	1636.51	9440.18	15.00	15.00	0

$$\theta(\partial A_1 \cap \partial A_2 = \phi) = 1 - \theta(\partial A_1 \cap \partial A_2 = \neg\phi) = 0.238$$

Among above computations, parameter d is the distance from the point '*21*' to the boundary '*11-12*' of A_1, equal to 6.58m; σ_1 denotes the standard error of the point '*21*', equal to $\sqrt{2}\times 15$=21.21m; σ_2 the standard error of the projection point, which can be computed using the error propagation expression derived by Li et al. (1995), equal to $\sqrt{2}\times 11.12$=15.73m. Further, the relative possibility is

$$\varphi(\gamma = '\mathit{covers}'|'\mathit{contains}') = 3.209$$

For iii), we may calculate the relative possibility using a similar approach to ii). And in this case, formula (22) can be reduced to

$$\varphi(\gamma = '\mathit{overlap}'|'\mathit{contains}') = \frac{\theta(A_2 - A_1 = \neg\phi)\theta(\partial A_1 \cap \partial A_2 = \neg\phi)}{\theta(A_2 - A_1 = \phi)\theta(\partial A_1 \cap \partial A_2 = \phi)}$$

Here, $'A_2 - A_1 = \neg\phi'$ means that there exists at least one point of A_2 outside A_1, while $'A_2 - A_1 = \phi'$ means that all of points of A_2 falls inside A_1 or on the boundary of A_1. As for Figure 6, it only needs to determine whether the point '21' is inside A_1 or outside. Their possibilities are computed as follows:

$$\theta(A_2 - A_1 = \neg\phi) = \exp(-\tfrac{(d+\sigma_1)^2}{\sigma_1^2})\times\exp(-\tfrac{(d+\sigma_2)^2}{\sigma_2^2}) = 0.140$$

$$\theta(A_2 - A_1 = \phi) = 1 - \theta(A_2 - A_1 = \neg\phi) = 0.860$$

Thus, the relative possibility is

$$\varphi(\gamma = '\mathit{overlap}'|'\mathit{contains}') = \frac{0.140\times 0.762}{0.860\times 0.238} = 0.521$$

By comparisons of the three relative possibilities, one can make the decision that the true relation between A_1 and A_2 is '*covers*'.

VI. DISCUSSIONS AND CONCLUSIONS

In this paper, positional uncertainty is analyzed and modeled statistically. With this model we make a detailed investigation on the effect of positional uncertainty on topological relations. A relative possibility-based approach for the determination of uncertain relations is proposed. A simple example is given for the illustration of the approach presented.

In this approach, the following new aspects are provided:

- Extended application of 4ID model presented by Chen and Deng (2003), where it is used for formal description of topological relation under certainty, while is now adapted under uncertainty.
- Basic possibility functions are derived, which is a bridge of positional uncertainty propagating to topological relation uncertainty.
- The approach presented can be simplified for use if epsilon band of probability distribution is taken for modeling location uncertainty of region boundary. And, the approach is also suitable for complex objects.

Further work is to concentrate on how to process topological inconsistency between neighboring objects. It possibly needs to generate a new common boundary.

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REFERENCES

[1] Burrough, P.A. and A.U. Frank, 1996. *Geographic objects with indeterminate boundaries*. Taylor and Francis, London, pp 345.

[2] Chen, J., C.M. Li, Z.L. Li, 2001. A Voronoi-based 9-intersection model for spatial relations. *International Journal of Geographical Information Science*, 15(3): 201-220

[3] Chen, X., 1996. Spatial relationships between uncertain sets. *International Archives of Photogrammetry and Remote Sensing*, XVIII congress. Vienna, 105-110.

[4] Chen, X. and M. Deng, 2003. Complete description of topological relations between spatial regions. In: Shi, W.Z., M. Goodchild, and P. Fisher (eds.), *The 2nd International Symposium on Spatial Data Quality*. HongKong, 51-60.

[5] Clementini, E. and P. Di Felice, 1995. A comparison of methods for representing topological relationships. *Information Systems*, 20(3): 149-178.

[6] Clementini, E. and P. Di Felice, 1996. An algebraic model for spatial objects with indeterminate boundaries. In: Burrough, P. and A. Frank (eds.), *Geographic Objects with Indeterminate Boundaries*, Taylor & Francis, London, 155-169.

[7] Cohn, A. and N. Gotts, 1996. The 'egg-yolk' representation of regions with indeterminate boundaries. In: Burrough, P. and A. Frank (eds.), *Geographic objects with Indeterminate Boundaries*, Taylor and Francis, London, 171-188.

[8] Egenhofer, M.J. and R. Franzosa, 1991. Point-set topological spatial relations. *International Journal of Geographical Information Systems*, 5(2): 161-174.

[9] Egenhofer, M.J., E. Clementini, and P. Di Felice, 1994. Topological relations between regions with holes. *International Journal of Geographic Information Systems*, 8(2): 129-144.

[10] Egenhofer, M.J. and D. Mark, 1995. Modeling conceptual neighborhoods of topological line- region relations. *International Journal of Geographical Information Systems*, 9 (5): 555-565.

[11] Gong, P., X. Zheng, and J. Chen, 1995. Boundary uncertainties in digital maps: an experiment on digitalization errors. *Geographic Information Sciences*, 1(2): 65-72
[12] Goodchild, M.F. and S. Gopal, 1989. *Accuracy of Spatial Databases*. Tayloy & Francis, London, 107-113.
[13] Guptill, S.C. and J.L. Morrison, 1995. *Elements of Spatial Data Quality*. Elsevier Scientific, Oxford.
[14] Leung, Y. and J.P. Yan, 1997. Point-in-polygon analysis under certainty and uncertainty. *GeoInformatica*, 1(1): 93-114.
[15] Leung, Y. and J.P. Yan, 1998. A locational error model for spatial features. *International Journal of Geographical Information Sciences*, 12(6): 607-620.
[16] Li, D.R. M.Y. Peng and J.Q. Zhang, 1995. Modeling positional uncertainty of line primitives in GIS, *Journal of WTUSM*, 21(4): 283-288.
[17] Li, Z.L., Y.L. Li and Y.Q. Chen, 2000. Basic topological models for spatial entities in 3-dimensional space. *GeoInformatica*, 4(4): 419-433.
[18] Li, Z.L., R.L. Zhao and J. Chen, 2002. A Voronoi-based spatial algebra for spatial relations. *Progress in Natural Science*, 12(7): 528-536.
[19] Liu, W.B., H.L. Dai, and P.L. Xu, 1998. Models of positional error donut for planar polygon in GIS. *Acta Geogdaetica et Cartographica Sinica*, 27 (4): 338-344.
[20] Molenaar, M., 1998. *An Introduction to the Theory of Spatial Object Modeling*. Taylor & Francis, London, pp.245.
[21] Shi, W.Z., 1994. *Modelling positional and thematic uncertainty in integration of GIS and remote sensing*. ITC Publication, Enschede, number 22, pp 147.
[22] Shi, W.Z. and W.B. Liu, 2000. A stochastic process-based model for positional error of line segments in GIS. *International Journal of Geographic Information Sciences*, 14 (1): 51-66.
[23] Winter, S., 2000. Uncertain topological relations between imprecision regions. *International Journal of Geographical Information Sciences*, 14 (5): 411- 430.

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