THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
UGEB2530B/C: Games & Strategic Thinking 2022-2023 Term 2
Homework Assignment 1
Due Date: 6 March, 2023 (Monday) before 11:59 PM

I declare that the assignment here submitted is original except for source material explicitly acknowledged, the piece of work, or a part of the piece of work has not been submitted for more than one purpose (i.e. to satisfy the requirements in two different courses) without declaration, and that the submitted soft copy with details listed in the "Submission Details" is identical to the hard copy, if any, which has been submitted. I also acknowledge that I am aware of University policy and regulations on honesty in academic work, and of the disciplinary guidelines and procedures applicable to breaches of such policy and regulations, as contained on the University website https://www.cuhk.edu.hk/policy/academichonesty/

It is also understood that assignments without a properly signed declaration by the student concerned will not be graded by the course teacher.

Signature ____________________ Date ____________________

General Regulations

- All assignments will be submitted and graded on Gradescope. You can view your grades and submit regrade requests there as well. For submitting your PDF homework on Gradescope, here are a few tips.

  Where is Gradescope?

  Do the following:

  1. Go to 2022R2 Games and Strategic Thinking (UGEB2530B)

  or

  Go to 2022R2 Games and Strategic Thinking (UGEB2530C)

  2. Choose Tools in the left-hand column
  3. Scroll down to the bottom of the page
  4. The green Gradescope icon will be there

- Late assignments will receive a grade of 0.

- Write your COMPLETE name and student ID number legibly on the cover sheet (otherwise we will not take any responsibility for your assignments). Please write your answers using a black or blue pen, NOT any other color or a pencil.

For the declaration sheet:

Either

Use the attached file, sign and date the statement of Academic Honesty, convert it into a PDF and submit it with your homework assignments via Gradescope.

Or

Write your name on the first page of your submitted homework, and simply write out the sentence "I have read the university regulations."

- Write your solutions on A4 white paper. Please do not use any colored paper and make sure that your written solutions are a suitable size (easily read). Failure to comply with these instructions will result in a 10-point deduction).

- Show all work for full credit. In most cases, a correct answer with no supporting work will NOT receive full credit. What you write down and how you write it are the most important means of your answers getting good marks on this homework. Neatness and organization are also essential.

Please attempt to solve all the problems. Your solutions for problems 1 - 9 are to be submitted.

We strongly recommended that you study Extra Exercises 1 - 9, though you are not required to submit their solutions. Suggested solutions for all the problems will be provided.

1. Evaluate the following matrix products.

(a) $\begin{pmatrix} 0 & 1 & 2 \end{pmatrix} \begin{pmatrix} 2 \\ 4 \\ 6 \end{pmatrix}$ (c) $\begin{pmatrix} 0 & 2 & 4 \\ 1 & 3 & 6 \end{pmatrix} \begin{pmatrix} 6 & 1 \\ 4 & 3 \\ 2 & 5 \end{pmatrix}$

(b) $\begin{pmatrix} 0 & 1 & 2 \\ 3 & 4 & 5 \end{pmatrix} \begin{pmatrix} 2 \\ 4 \\ 6 \end{pmatrix}$ (d) $\begin{pmatrix} 6 \\ 4 \\ 2 \end{pmatrix} \begin{pmatrix} 1 & 3 \end{pmatrix}$

2. Suppose a die and 2 coins are tossed together. Let x be the number obtained from the dice and y be the number of heads shown among the coins.

(a) Fill in the blanks in the following tables:

x	1	2	3	4	5	6
$P(\text{getting } x)$						

y	0	1	2
$P(\text{getting } y)$			

(b) Using part (a), fill in the blanks in the following table:

z	1	2	3	4	5	6	7	8
$P(x+y=z)$								

(c) Now, evaluate the expected value of $x + y$.

3. In a rock-paper-scissors game, the loser pays the total number of fingers in each two-gesture round to the winner. The payoff is 0 if there is a draw.

(a) Write down the game matrix (payoff of player 1) of the game. (Use rock, paper, scissors, as the order of strategies.)

(b) Suppose player 1 uses $(0.2, 0.4, 0.4)$ and player 2 uses $(0.3, 0.5, 0.2)$. Find the expected payoff of player 1.

(c) If player 1 uses $(0.2, 0.4, 0.4)$, what is the best strategy for player 2.

(d) If player 2 uses $(0.3, 0.5, 0.2)$, what is the best strategy for player 1.

(e) By considering equalizing strategies, find a Nash equilibrium and the value of the game.

4. In a game, two players call out one of the numbers 1, 2, or 3 simultaneously. Let S be the sum of the two numbers. If S is even, then player 2 pays S dollars to player 1. If S is odd, then player 1 pays S dollars to player 2.

(a) Write down the game matrix for the payoff of player 1.

(b) Find the expected payoff of player 1 if player 1 calls out the numbers 1, 2, 3 with probabilities 0.5, 0.3, 0.2, respectively, and player 2 calls out the numbers 1, 2, 3 with probabilities 0.1, 0.4, 0.5, respectively.

(c) Suppose player 2 calls out the numbers 1,2,3 with probabilities 0.1, 0.4, 0.5 respectively. What is the best strategy for player 1 and what is his expected payoff if he uses this strategy?

5. For each of the following game matrices, determine whether there is a saddle point. Copy the game matrix and circle all saddle points of the matrix if there are any.

(a) $\begin{pmatrix} -1 & -4 & 5 & -2 \\ -3 & 5 & -1 & 0 \\ 2 & 4 & -1 & 3 \end{pmatrix}$

(b) $\begin{pmatrix} -3 & 7 & -3 & 0 \\ 0 & -4 & -1 & -3 \\ 2 & 4 & 6 & 4 \\ -2 & 2 & 3 & 1 \end{pmatrix}$

(c) $\begin{pmatrix} 4 & 2 & 5 & 2 \\ 2 & 1 & -1 & -20 \\ 3 & 2 & 4 & 2 \\ -20 & 0 & 18 & 1 \end{pmatrix}$

6. Solve the zero sum games, that is, find a maximin strategy for the row player, a minimax strategy for the column player and the value of the game for the following game matrices.

(a) $\begin{pmatrix} 3 & -1 \\ 0 & 1 \end{pmatrix}$

(b) $\begin{pmatrix} 1 & 5 \\ 4 & 2 \end{pmatrix}$

7. Solve the zero sum games for the following game matrices.

(a) $\begin{pmatrix} 1 & -1 & 3 \\ 3 & 5 & -3 \end{pmatrix}$

(b) $\begin{pmatrix} -1 & 6 \\ 0 & 4 \\ 2 & 3 \\ 3 & 1 \end{pmatrix}$

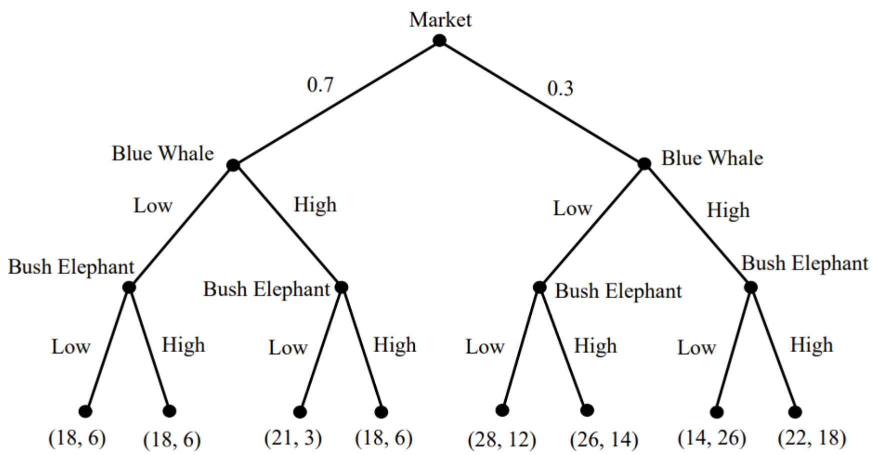
8. Solve the zero sum game for game matrix

$$\begin{pmatrix} 5 & 3 & 8 & 1 \\ 2 & 3 & 5 & 10 \\ 7 & 5 & 6 & 2 \\ 6 & 4 & 3 & 1 \end{pmatrix}$$

9. Two tech companies, Blue Whale (as Row) and Bush Elephant (as Col), have to decide whether they should produce a high quality but expensive digital thermometer or a lower quality but more affordable one. There's a 70% chance that the market will for both products will turn out to be small and a 30% chance that it will be large.

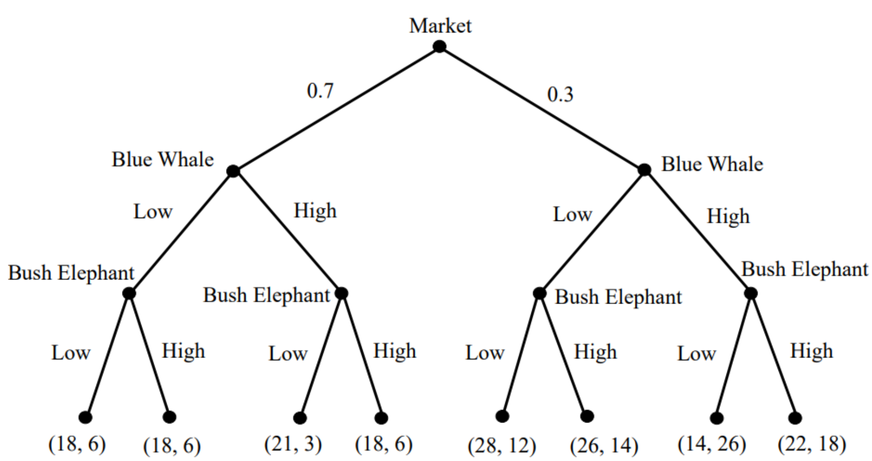
(a) Suppose both companies have no additional information about the market and don't know each other's decision.

(i) Draw dotted lines on the following game tree to describe the information sets.



(ii) Find the optimal strategies for both companies and their corresponding payoffs.

(b) Suppose both companies have no additional information about the market, but Bush Elephant knows Blue Whale's decision.



Draw dotted lines on the following game tree to describe the information sets.

Extra exercises

E1. (a) Suppose

$$A = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 2 \\ 3 & 0 \end{pmatrix}, \quad C = \begin{pmatrix} \frac{1}{3} \\ \frac{2}{3} \end{pmatrix}.$$

Compute AB, BC and ABC.

(b) Suppose

$$D = \begin{pmatrix} p & 1-p \end{pmatrix}.$$

Find the value of p such that the entries of DB are all equal.

(c) Suppose

$$E = \begin{pmatrix} q \\ 1-q \end{pmatrix}.$$

Find the value of q such that the entries of BE are all equal.

E2. Let

$$A = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, B = \begin{pmatrix} 3 & -2 \end{pmatrix}, C = \begin{pmatrix} 1 & -1 \\ 0 & 2 \end{pmatrix} \text{ and } D = \begin{pmatrix} 1 & 0 \\ 1 & 3 \end{pmatrix}.$$

Compute the following. Write "undefined" for those which are not defined.

(a) AB
(b) $B^T A^T$ and CAB
(c) $C^T - D^2$
(d) AD

E3. An **unfair** die with six sides is tossed. Let x be the number that comes up. The probability of obtaining different values of x are shown in the following table:

x	$P(\text{getting } x)$
1	0.3
2	0.2
3	0.2
4	0.1
5	0.1
6	0.1

(a) Find the expected value of x.
(b) Suppose the unfair die is tossed with a fair die. Find the probability that the sum of the two numbers shown is 9.

E4. Suppose a family has 5 children and the probability of having a girl is $\frac{1}{2}$. Find the probability that the family has the following children:

(a) Exactly 2 girls and 3 boys
(b) Exactly 3 girls and 2 boys

(c) No girls

(d) No boys

(e) At least 3 boys

(f) No more than 4 girls

E5. Suppose a die with six sides is tossed. Let x be the number that comes up. Evaluate the expected values of $x + 1$ and x^2.

E6. For each of the following payoffs matrices (2-person, zero-sum, simultaneous games), circle all the saddle point(s) (if any).

(a)

$$\begin{pmatrix} -3 & 5 & -7 & 0 \\ -1 & -3 & -5 & -2 \\ 2 & 4 & -6 & 1 \end{pmatrix}$$

(b)

$$\begin{pmatrix} 5 & -1 & -3 & 0 \\ 2 & -2 & 3 & 1 \\ -4 & 0 & -1 & -3 \\ 3 & 1 & 6 & 4 \end{pmatrix}$$

E7. For the following 2-person, zero-sum, simultaneous game, find a mixed Nash equilibrium and the value of the game:

	C1	C2	C3	C4
R1	4	3	2	-1
R2	2	5	0	-2
R3	-1	2	0	7
R4	3	0	1	-2

E8. Consider the following game. **Harden** and **James** are both in room 1 and **Curry** is in room 2. **Harden** first flips a coin. **James** goes to room 2 afterward and tells **Curry** $Head$ or $Tail$. **Curry** then chooses $Believe$, Not or $Fold$.

- **James** pays **Curry** \$10 if what **James** said matches the flipped coin and **Curry** chooses $Believe$
- **James** pays **Curry** \$20 if what **James** said doesn't match the flipped coin and **Curry** chooses Not
- **Curry** pays **James** \$20 if what **James** said matches the flipped coin and **Curry** chooses Not

- **Curry** pays **James** \$10 if what **James** said doesn't match the flipped coin and **Curry** chooses *Believe*
- **Curry** pays **James** \$2 if **Curry** chooses *Fold*

Draw its game tree using **Harden**, **James**, **Curry** as player $1, 2, 3$, respectively. Include **James**' payoff for each outcome.

E9. For each of the following games,

(i) Find all the pure Nash equilibrium(s) if they/it exist(s). If not, find a mixed Nash equilibrium.

(ii) Find the value of the game.

(a)

	C1	C2
R1	3	0
R2	-1	1

(b)

	C1	C2	C3	C4
R1	1	5	-2	-3
R2	-1	-3	3	6

(c)

	C1	C2	C3
R1	-1	3	4
R2	1	1	2
R3	-2	5	5
R4	4	0	1

(d)

	C1	C2	C3	C4
R1	0	-2	3	-4
R2	-3	-2	0	5
R3	2	0	1	-3
R4	1	-1	-2	-4

(e)

	C1	C2	C3	C4	C5
R1	1	3	2	7	4
R2	3	4	1	5	6
R3	6	5	7	6	5
R4	2	0	6	3	1