THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MATH3280 Introductory Probability 2022-2023 Term 2
Homework Assignment 3
Due Date: 6 March, 2021 (Monday)

I declare that the assignment here submitted is original except for source material explicitly acknowledged, the piece of work, or a part of the piece of work has not been submitted for more than one purpose (i.e. to satisfy the requirements in two different courses) without declaration, and that the submitted soft copy with details listed in the "Submission Details" is identical to the hard copy, if any, which has been submitted. I also acknowledge that I am aware of University policy and regulations on honesty in academic work, and of the disciplinary guidelines and procedures applicable to breaches of such policy and regulations, as contained on the University website https://www.cuhk.edu.hk/policy/academichonesty/

It is also understood that assignments without a properly signed declaration by the student concerned will not be graded by the course teacher.

Signature ______________________ Date ______________________

General Regulations

- All assignments will be submitted and graded on Gradescope. You can view your grades and submit regrade requests there as well. For submitting your PDF homework on Gradescope, here are a few tips.

  Where is Gradescope?

  Do the following:

  1. Go to 2022R2 Introductory Probability (MATH3280B)
  2. Choose Tools in the left-hand column
  3. Scroll down to the bottom of the page
  4. The green Gradescope icon will be there

- Late assignments will receive a grade of 0.

- Write your COMPLETE name and student ID number legibly on the cover sheet (otherwise we will not take any responsibility for your assignments). Please write your answers using a black or blue pen, NOT any other color or a pencil.

  For the declaration sheet:

  Either

Use the attached file, sign and date the statement of Academic Honesty, convert it into a PDF and submit it with your homework assignments via Gradescope.

Or

Write your name on the first page of your submitted homework, and simply write out the sentence "I have read the university regulations."

- Late assignments will receive a grade of 0.
- Write your COMPLETE name and student ID number legibly on the cover sheet (otherwise we will not take any responsibility for your assignments). Please write your answers using a black or blue pen, NOT any other color or a pencil.
- Write your solutions on A4 white paper. Please do not use any colored paper and make sure that your written solutions are a suitable size (easily read). Failure to comply with these instructions will result in a 10-point deduction.
- Show all work for full credit. In most cases, a correct answer with no supporting work will NOT receive full credit. What you write down and how you write it are the most important means of your answers getting good marks on this homework. Neatness and organization are also essential.

1. Two balls are chosen randomly from an urn containing 8 white, 4 black, and 2 orange balls. Suppose that we win \$2 for each black ball selected and we lose \$1 for each white ball selected. Let X denote our winnings.

   (a) What are the possible values of X, and what are the probabilities associated with each value?

   (b) If we play the game 100 times, and to play every time we have to pay \$2 as table money, what is the amount we should expect to get?

   (c) Is the game fair? Explain.

2. A salesman has scheduled two appointments to sell encyclopedias. His first appointment will lead to a sale with probability .3, and his second will lead independently to a sale with probability .6. Any sale made is equally likely to be either for the deluxe model, which costs \$1000, or the standard model, which costs \$500. Determine the probability mass function of X, the total dollar value of all sales.

3. A total of 4 buses carrying 148 students from a school arrive at a football stadium. The buses carry, respectively, 40, 33, 25, and 50 students. One of the students is randomly selected. Let X be the number of students that were on the bus carrying this randomly selected student. One of the 4 bus drivers is also randomly selected. Let Y be the number of students on her bus. Determine $E[X]$, $E[Y]$, $\text{Var}(X)$ and $\text{Var}(Y)$.

4. Suppose that the distribution function of X is given by

$$F(b) = \begin{cases} 0 & b < 0; \\ b/4 & 0 \le b < 1; \\ 1/2 + (b-1)/4 & 1 \le b < 2; \\ 11/12 & 2 \le b < 3; \\ 1 & 3 \le b \end{cases}$$

   (a) Find $P(X = i),\ i = 1, 2, 3.$

   (b) Find $P\left(\frac{1}{2} < X < \frac{3}{2}\right)$.

5. When coin 1 is flipped, it lands on heads with probability .4; when coin 2 is flipped, it lands on heads with probability .7. One of these coins is randomly chosen and flipped 10 times.

   (a) What is the probability that the coin lands on heads for exactly 7 of the 10 flips?

   (b) Given that the first of these ten flips lands on heads, what is the conditional probability that exactly 7 of the 10 flips land on heads?

6. An interviewer is given a list of people she can interview. If the interviewer needs to interview 5 people, and if each person (independently) agrees to be interviewed with probability 2/3,

(a) For part (a), what is the probability that her list of people will enable her to obtain her necessary number of interviews if the list consists of (a) 5 people and (b) 8 people?

(b) For part (b), what is the probability that the interviewer will speak to exactly (c) 6 people and (d) 7 people on the list?

Note that there are four subproblems in this question.

7. An absentminded worker does not remember which of his 12 keys will open his office door. If he tries them at random and with replacement:

   (a) On average, how many keys should he try before his door opens?

   (b) What is the probability that he opens his office door after only three tries?

8. Let X be such that

$$P(X = 1) = p = 1 - P(X = -1)$$

   Find $c \neq 1$ such that $E[c^X] = 1$.

9. Let X be a binomial random variable with parameters n and p. Show that

$$E\left[\frac{1}{X+1}\right] = \frac{1-(1-p)^{n+1}}{(n+1)p}.$$

10. Let X be a Poisson random variable with parameter λ. What value of λ maximizes $P(X = k)$, $k \geq 0$?

11. Show that X is a Poisson random variable with parameter λ, then

$$E\left[X^n\right] = \lambda E\left[(X+1)^{n-1}\right]$$

   Now use this result to compute $E\left[X^3\right]$.

12. Let X be a negative binomial random variable with parameters r and p and let Y be a binomial random variable with parameters n and p. Show that

$$P(X > n) = P(Y < r).$$