

THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MATH3280 Introductory Probability 2022-2023 Term 2
Homework Assignment 2
Due Date: 20 February, 2023 (Monday)

I declare that the assignment here submitted is original except for source material explicitly acknowledged, the piece of work, or a part of the piece of work has not been submitted for more than one purpose (i.e. to satisfy the requirements in two different courses) without declaration, and that the submitted soft copy with details listed in the “Submission Details” is identical to the hard copy, if any, which has been submitted. I also acknowledge that I am aware of University policy and regulations on honesty in academic work, and of the disciplinary guidelines and procedures applicable to breaches of such policy and regulations, as contained on the University website <https://www.cuhk.edu.hk/policy/academichonesty/>

It is also understood that assignments without a properly signed declaration by the student concerned will not be graded by the course teacher.

Signature

Date

General Regulations

- All assignments will be submitted and graded on Gradescope. You can view your grades and submit regrade requests there as well. For submitting your PDF homework on Gradescope, [here are a few tips](#).

Where is Gradescope?

Do the following:

1. Go to 2022R2 Introductory Probability (MATH3280B)
 2. Choose Tools in the left-hand column
 3. Scroll down to the bottom of the page
 4. The green Gradescope icon will be there
- Late assignments will receive a grade of 0.
 - Write your COMPLETE name and student ID number legibly on the cover sheet (otherwise we will not take any responsibility for your assignments). Please write your answers using a black or blue pen, NOT any other color or a pencil.

For the declaration sheet:

Either

Use the attached file, sign and date the statement of Academic Honesty, convert it into a PDF and submit it with your homework assignments via Gradescope.

Or

Write your name on the first page of your submitted homework, and simply write out the sentence “I have read the university regulations.”

- Write your solutions on A4 white paper. Please do not use any colored paper and make sure that your written solutions are a suitable size (easily read). Failure to comply with these instructions will result in a 10-point deduction.
- Show all work for full credit. In most cases, a correct answer with no supporting work will NOT receive full credit. What you write down and how you write it are the most important means of your answers getting good marks on this homework. Neatness and organization are also essential.

- If two fair dice are rolled, what is the conditional probability that the first one lands on 6 given that the sum of the dice is i ? Compute for all values of i between 2 and 12.
- What is the probability that at least one of a pair of fair dice lands on 6, given that the sum of the dice is i , $i = 2, 3, \dots, 12$?
- Two cards are randomly chosen without replacement from an ordinary deck of 52 cards. Let B be the event that both cards are aces, let A_h be the event that the ace of hearts is chosen, and let A be the event that at least one ace is chosen. Find
 - $P(B|A_h)$
 - $P(B|A)$
- A total of 500 married working couples were polled about their annual salaries, with the following information resulting:

		Husband	
		Less than \$25,000	More than \$25,000
Wife	Less than \$25,000	212	198
	More than \$25,000	36	54

For instance, in 36 of the couples, the wife earned more and the husband earned less than \$25,000. If one of the couples is randomly chosen, what is

- the probability that the husband earns less than \$25,000?
 - the conditional probability that the wife earns more than \$25,000 given that the husband earns more than this amount?
 - the conditional probability that the wife earns more than \$25,000 given that the husband earns less than this amount?
- E and F play a series of games. Each game is independently won by E with probability p and by F with probability $1 - p$. They stop when the total number of wins of one of the players is two greater than that of the other player. The player with the greater number of total wins is declared the winner of the series.
 - Find the probability that a total of 4 games are played.
 - Find the probability that E is the winner of the series.
 - Let $A \subset B$. Express the following probabilities as simply as possible:
 - $P(A|B)$;
 - $P(A|B^c)$;
 - $P(B|A)$;
 - $P(B|A^c)$.

7. Mary is performing independent rolls of two dice.

- (a) What is the probability that the sum of two dice is 4?
- (b) What is the probability that the { sum of two dice is 4 } appears before the outcome { sum of two dice is 8 }? (*Hint:* Let A_n be the event that no 4 or 8 appears on the first $n - 1$ rolls and a 4 appears on the n th roll. Find $P(\cup_{n=1}^{\infty} A_n)$.)

8. Independent trials that result in a success with probability q and a failure with probability $1 - q$ are called Bernoulli trials. Let Q_n denote the probability that n Bernoulli trials result in an even number of successes (0 being considered an even number). Show that

$$Q_n = q(1 - Q_{n-1}) + (1 - q)Q_{n-1}, n \geq 1$$

and use this formula to prove (by induction) that

$$Q_n = \frac{1 + (1 - 2q)^n}{2}$$

9. Let C_n denote the probability that no run of 3 consecutive heads appears in n tosses of a fair coin. Show that

$$C_n = \frac{1}{2}C_{n-1} + \frac{1}{4}C_{n-2} + \frac{1}{8}C_{n-3}$$

$$C_0 = C_1 = C_2 = 1$$

Find C_8 . (*Hint:* condition on the first trial)

10. For a fixed event B , show that the collection $P(A|B)$, defined for all events A , satisfies the three conditions for a probability. Conclude from this that

$$P(A|B) = P(A|B \cap C)P(C|B) + P(A|B \cap C^c)P(C^c|B)$$

Then directly verify the preceding equation.