1. Recall the notions of *real part*, *imaginary part*, *conjugate* and *modulus*, introduced in the handout *Basic algebraic results on complex numbers 'beyond school mathematics'*:

   Let z be a complex number. Denote the real part and the imaginary part of z by $\mathsf{Re}(z)$, $\mathsf{Im}(z)$ respectively. (So $z = \mathsf{Re}(z) + i\mathsf{Im}(z)$.)

   (a) *The* **complex conjugate** *of z is defined to be the complex number $\mathsf{Re}(z) - i\mathsf{Im}(z)$. It is denoted by $\overline{z}$.*

   (b) *The* **modulus** *of z, denoted by $|z|$, is defined to be the non-negative real number $\sqrt{(\mathsf{Re}(z))^2 + (\mathsf{Im}(z))^2}$.*

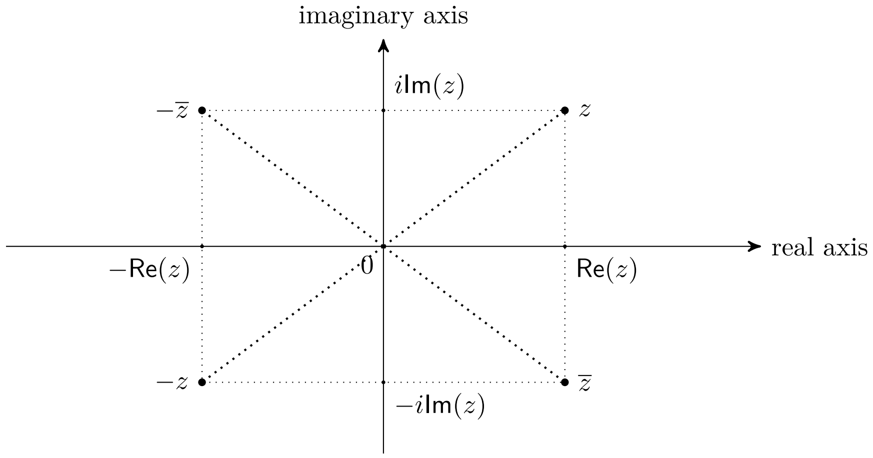
2. **Complex numbers and Euclidean plane geometry.**

   Each complex number z may be thought of as the point on the 'infinite plane', known in this context as the **Argand plane**, with rectangular coordinates $(\mathsf{Re}(z), \mathsf{Im}(z))$.

   We refer to the point $(\mathsf{Re}(z), \mathsf{Im}(z))$ simply as '*the point z*'.

   $|z|$ is the (Euclidean) distance between 0 and z. It is also the length of the vector 'pointing from 0 to z'.

   $\overline{z}$ is the point obtained by reflecting the point z with respect to the 'real axis'. (How about $-z$, $-\overline{z}$?)



3. **'Dictionary' between the algebra of complex numberes and coordinate geometry.**

   The Argand plane provides a 'dictionary' between the algebra of complex numbers and coordinate geometry.

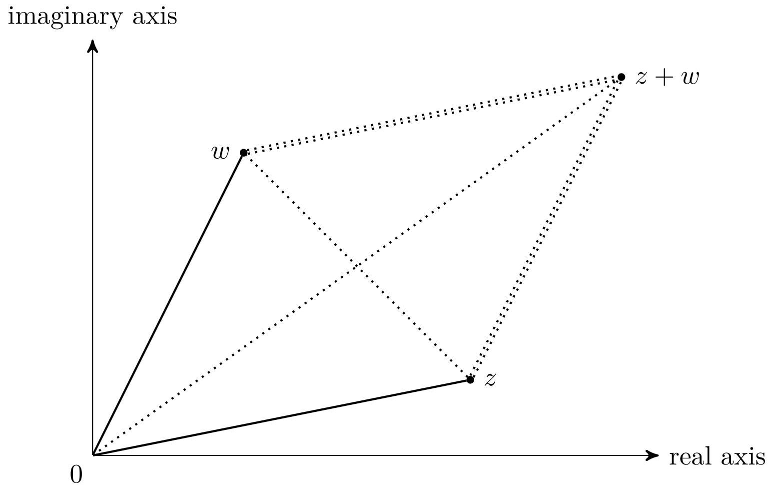
   The basic definitions and results concerned with the arithmetic of complex numbers, introduced in the handout *Basic algebraic results on complex numbers 'beyond school mathematics'*, can be given geometric interpretation on the Argand plane.

   The basic definitions, such as collinearity, perpendicularity and parallelism, and results in Euclidean plane geometry can be expressed in terms of the algebra of complex numbers.

4. **Geometric interpretation of addition and Triangle Inequality.**

Suppose z, w be distinct non-zero complex numbers. Then the points 0, z, w, $z+w$ are the vertices of a parallelogram in which:

- *the line segment joining 0 and z is parallel to the line segment joining w and $z+w$,*
- *the line segment joining 0 and w is parallel to the line segment joining z and $z+w$, and*
- *the line segment joining z and w and the line segment joining 0 and $z+w$ are diagonals.*



Remark.

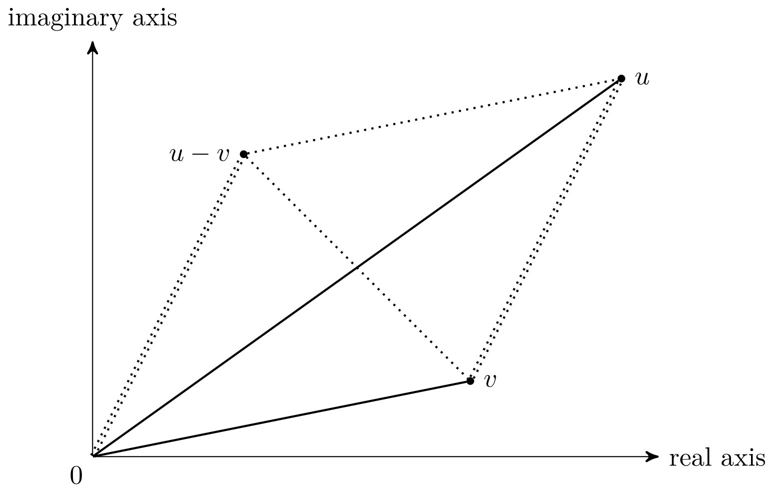
(a) We interpret 'adding z by w' as: translating the line segment joining 0 to w to the line segment joining z and $z+w$ via parallelism.

(b) The inequality $|z+w| \leq |z| + |w|$ looks 'obvious': we expect the line segment joining 0 and $z+w$ to be no lengthier than the path joining 0 to $z+w$ made up by the two line segments, respectively joining 0 and z, and joining z and $z+w$.

5. **Geometric interpretation of subtraction.**

Suppose u, v are distinct non-zero complex numbers. The point $u-v$ is the number so that 0, v, $u-v$, v forms the vertices of a parallelogram in which:

- *the line segment joining 0 and $u-v$ is parallel to the line segment joining v and u,*
- *the line segment joining 0 and v is parallel to the line segment joining $u-v$ and u, and*
- *the line segment joining 0 and u and the line segment joining v and $u-v$ are diagonals.*



Remarks.

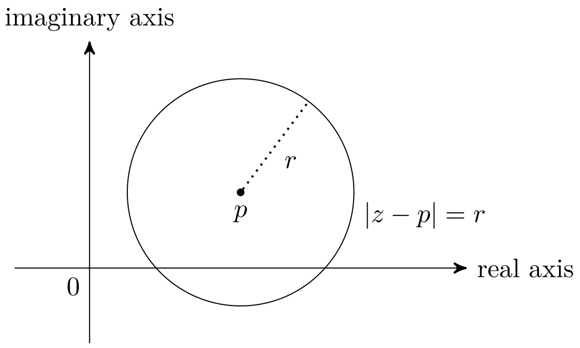
(a) We interpret 'subtracting u by v' as: translating the line segment joinint v to u to the line segment joining 0 and $u-v$ via parallelism.

(b) $|u-v|$ is the (Euclidean) distance between u and v.

6. **Circles and discs.**

Suppose p is a complex number, and r is a non-negative real number.

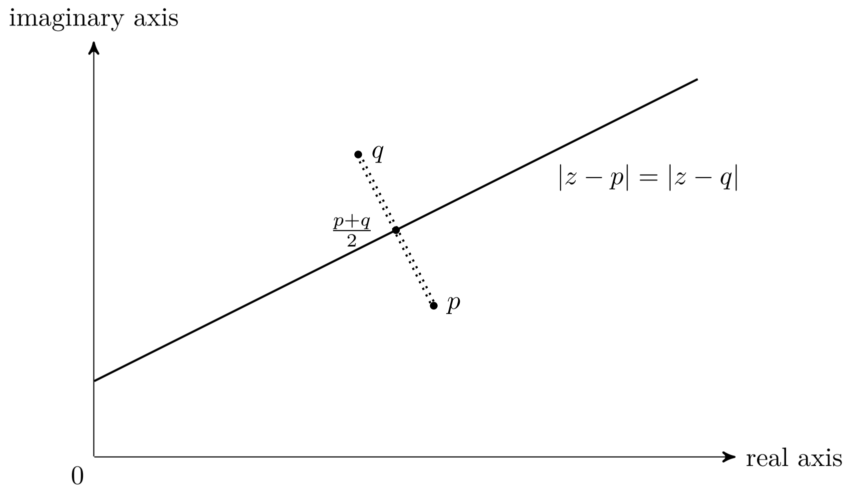
- *The equation $|z - p| = r$ with complex unknown z describes the circle with centre p and of radius r.*
- *The inequality $|z - p| \leq r$ with complex unknown z describes the closed disc with centre p and of radius r.* When $r = 0$, this closed disc degenerates into the point p.
- *Suppose $r > 0$. Then the inequality $|z - p| < r$ with complex unknown z describes the open disc with centre p and of radius r.*



7. **Straight line as perpendicular bisector.**

Suppose p, q are complex numbers.

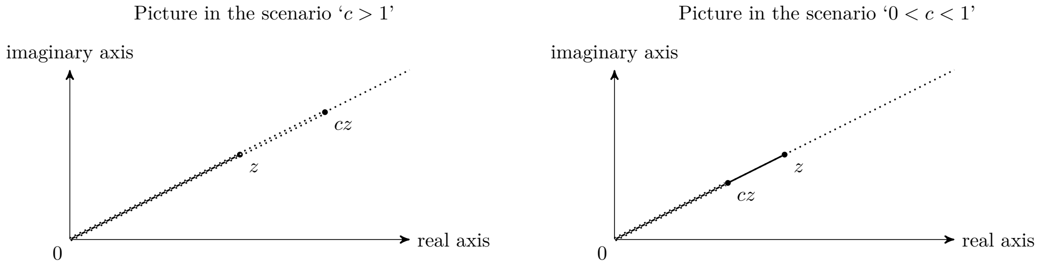
The equation $|z - p| = |z - q|$ with complex unknown z describes the straight line which is the perpendicular bisector for the line segment joining p, q.



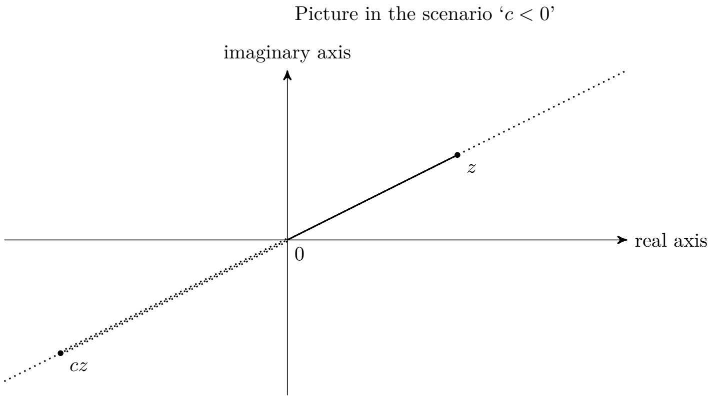
8. **Geometric interpretation of multiplication by real numbers.**

Suppose z is a non-zero complex number, and c is a real number.

(a) *Suppose $c > 0$. Then cz is the complex number on the same half line starting from 0 and joining z so that the distance between 0 and cz is $c|z|$.*



(b) *Suppose $c < 0$. Then cz is the complex number on the same half line starting from 0 and joining $-z$ so that the distance between 0 and cz is $-c|z|$.*



9. **Geometric interpretation of multiplication by i.**

Suppose z be a non-zero complex number.

Then the points z, iz, $-z$, $-iz$ are the four vertices of a square in which the line segment joining $z, -z$ and the line segment joining $iz, -iz$ are the diagonals.

The latter is obtained by rotating the former about the point 0 by $\pi/2$ radians, so that iz is obtained from z, and $-iz$ is obtained from $-z$.