Integrals of the day: Part 2

1. Compute
$$\int_0^{\frac{\pi}{2}} \frac{1}{1 + (\tan x)^{\sqrt{5}}} dx.$$

Hint: Substitute $u = \frac{\pi}{2} - x$, and use symmetry.

2. Compute
$$\int \cos(\ln x) dx.$$

Hint: Integrate by parts twice, or use the substitution $u = \ln x$ to reduce to the more familiar integral $\int e^x \cos x \, dx$.

3. Compute
$$\int \frac{\sin x}{\sin x + \cos x} dx$$

Hint: Either let $I = \int \frac{\sin x}{\sin x + \cos x} dx$, $J = \int \frac{\cos x}{\sin x + \cos x} dx$ and compute both $I + J$ and $I - J$ (the latter can be computed using the substitution $u = \sin x + \cos x$), or write $\frac{\sin x}{\sin x + \cos x} = \frac{\tan x}{1 + \tan x}$ and substitute $u = \tan x$. Yet another way is t-substitution: substitute $t = \tan \frac{x}{2}$.

End