Math 1010C Term 1 2014
Supplementary exercises 3

The following exercises are not to be submitted, but they form an important part of the course, and you're advised to go through them carefully.

In supplementary exercise 2, we saw how one could find the absolute maximum / minimum of a continuous function on a closed and bounded interval. In the following, we will locate relative maximums / minimums of a function, and find the absolute maximum / minimum of a function on an unbounded interval (if it exists).

1. Find all critical points of the following functions on the indicated intervals. Determine whether these are relative maximums / minimums of the functions (they could be neither).
   (a) $f(x) = x^{1/3}(x-4)$, $\quad (-1, \infty)$
   (b) $g(x) = x\sqrt{8-x^2}$, $\quad (-2\sqrt{2}, 2\sqrt{2})$
   (c) $h(x) = x \ln x$, $\quad (0, \infty)$

2. For each of the following function,
   (i) Determine where the function is increasing, and where it is decreasing;
   (ii) Find all relative maximums / minimums of the function on $(-\infty, \infty)$;
   (iii) Determine whether any of these is an absolute extremum of the function on $(-\infty, \infty)$. (For this you will need to understand the behaviour of the function at $\pm\infty$.)
   (iv) Determine where the function is convex, and where it is concave;
   (v) Sketch the graph of the function.

   (a) $f(x) = x^3 - 12x - 5$
   (b) $g(x) = x^2(1-x^2)$
   (c) $h(x) = \dfrac{x}{x^2+1}$

3. For each of the functions and intervals in Question 1, determine whether the given function have an absolute maximum / minimum on the indicated intervals. (You'll have to understand the behaviour of these functions as x approaches the end-points of the intervals.) If yes, find the maximum / minimum values of the functions on the indicated intervals.

4. Determine whether the following functions have an absolute maximum / minimum on the indicated intervals. If yes, locate ALL points where the absolute maximum / minimum are achieved.
   (a) $f(x) = e^{2x} + e^{-x}$, $\quad [0, \infty)$
   (b) $g(x) = \dfrac{x^2-3}{x-2}$, $\quad (-\infty, 2)$
   (c) $h(x) = \dfrac{2x^2 - x^4}{x^4 - 2x^2 + 2}$, $\quad [-1, \infty)$

   (Credit: Many of the above functions are taken from Thomas' calculus, chapter 4.)