An integer programming based strategy for Asian-style futures arbitrage over the settlement period

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Abstract

An Asian-style futures is settled by an Asian-style settlement procedure, more specifically, it is settled against the arithmetic average of the underlying asset prices taken over the settlement period. In this paper, we propose a practical trading strategy based on an integer programming technique to exploit the mispricing opportunity of Asian-style index futures over the settlement period using a proxy of the underlying asset. The integer program can detect mispricing, construct an arbitrage portfolio by using the proxy and dynamically maintain the arbitrage portfolio. Hang Seng Index Futures (HSI Futures) of the Hong Kong market is used to test the trading strategy. The historical data of HSI Futures shows that there is a positive relationship between the magnitude of mispricing and the time to maturity over the settlement period. Moreover, our empirical findings show positive profitability of the trading strategy.

1 Introduction

An Asian-style futures is a futures contract with its settlement price calculated using an Asian-style settlement procedure, which is the arithmetic average of the underlying asset prices taken over the settlement period, typically the expiration day. Table 1 lists some exchanges and their futures that use an Asian-style settlement procedure.

In this paper, we propose a practical trading strategy to perform Asian-style index futures arbitrage over the settlement period using a proxy of the underlying asset. Arbitrage related activities on the expiration

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Exchange	Product	Settlement procedure
Australian Securities Exchange	S&P/ASX 200 VIX Futures	The arithmetic average of the underlying index values taken at every 30 seconds between 11:30 a.m. and 12:00 p.m. on the expiration day.
Eurex Exchange	STOXX Europe 600 Sector Index Futures	The arithmetic average of the underlying index values taken at every 15 seconds between 11:50 and 12:00 on the expiration day.
Euronext Amsterdam	AEX Index Futures	The arithmetic average of the underlying index values quotations taken at every one minute between 15:30 and 16:00 on the expiration day.
Madrid Stock Exchange	Futures on IBEX 35	The arithmetic average of the underlying index values taken at every one minute between 16:15 and 16:45 on the expiration day.
Hong Kong Exchanges and Clearing	Hang Seng Index Futures	The arithmetic average of the underlying index values taken at every five minutes on the expiration day.

Table 1: Exchanges and their Asian-style settlement procedure

day could result in unusual price movements and trading volumes of the underlying asset, which is generally referred to expiration day effect. Here we will report some of the existing studies on the expiration day effect in different markets from around the globe. In the U.S. market, Stoll and Whaley (1987) indicate abnormally high trading volumes concentrating within the "triple witching hour" and a significantly higher volatility among index constituent stocks on the expiration day as derivative contracts are settled against the closing price of the underlying index. In June 1987, the settlement price is changed to be the opening index of the day after the expiration day. Herbst and Maberly (1990) find that the change of settlement price shifts the high volatility effect to the opening. Chamberlain *et al.* (1989) study the Toronto market and they report abnormally large trading volumes and volatility on expiration days and price reversals following the expiration of derivative contracts. In the Swedish market, Alkebäck and Hagelin (2004) find significantly higher trading volumes of OMX Index constituent stocks on expiration days than on other days, where OMX Index futures are settled against the volume-weighted average index on the expiration day. Vipul (2005) reports that in the Indian market, prices of the underlying stocks are suppressed on the day before expiration of options and futures but bounce back on the day after expiration. In the Hong Kong market, Chow *et al.* (2003) show that there is a higher volatility and lower average five-minute returns of Hang Seng Index on expiration days than on other days. Fung and Yung (2009) find trading intensifies in terms of both volume and frequency concentrating around the quotation time marks, and order imbalance patterns on some expiration days. They also provide evidences indicating that arbitrage and direction-related trading activities are concentrated in large-capitalization stocks.

To take advantage of a mispricing opportunity, arbitrageurs need to take an offsetting arbitrage position in order to replicate price movements of the underlying asset. In a risk-free manner, one may employ a whole basket of constituent stocks, along with their corresponding weights. However, the profitability of such arbitrage strategy has been questionable in the sense that it is difficult to trade the exact weights and trade every constituent stock. Hence arbitrageurs usually use a sub-basket of the constituent stocks. Previous works have examined using constituent stocks to construct an arbitrage portfolio and their findings differ. Modest and Sundaresan (1983) find that with the existence of transaction cost, the price of index futures fluctuates within a band around its fair value without occurring any mispricing opportunity even for the most favorably situated arbitrageur. On the other hand, Yadav and Pope (1994) report magnitudes of mispricing often exceed the estimated transaction costs. Moreover, researchers report similar results under short sale constraints. Chung (1991), Chan and Chung (1993) and Fung and Draper (1999) find that short sale restrictions in cash markets hinder arbitrageurs from exploiting index futures underpricing and thus result in slower price adjustments. After the introduction of ETF, Richie *et al.* (2008) show that persistent mispricing opportunities do exist when using the SPDR ETF as a proxy for the S&P 500 index. They also find that insufficient volume sizes are the key limit of performing arbitrage.

While there are studies reporting the expiration day effect as well as examining index futures mispricing, this paper extends previous works by proposing a practical trading strategy for Asian-style futures arbitrage using a proxy of the underlying asset. Since the delta of an Asian-style futures decreases at each time mark where the price quotation is taken, arbitrageurs need to unwind the offsetting position at each time mark in order to track the decreasing delta. Our strategy can perform the unwinding automatically. In particular, the trading strategy is an integer program which can perform mispricing detection, arbitrage portfolio construction and dynamic maintenance of the arbitrage portfolio. Hang Seng Index Futures (HSI Futures) in the Hong Kong market will be used to test the profitability and effectiveness of the trading strategy. First back month contracts will be used as the proxy in the test.

The rest of the paper is organized as follows. In Section 2, we derive the fair value of an Asian-style futures during the settlement period. Section 3 introduces a practical trading strategy. Section 4 discusses factors that could erode the profitability of an arbitrage opportunity and suggests a solution. Section 5 summarizes the process of using the trading strategy. Section 6 describes the data of HSI Futures used for back-testing. In Section 7, we analyze the patterns of mispricing of the HSI Futures on the expiration day. Section 8 shows the empirical results of applying our trading strategy to perform Hang Seng Index Futures arbitrage. Section 9 concludes our findings.

2 Fair Value during Settlement Period

The fair value of a plain vanilla futures contract is given by the current underlying asset price multiplying the interest rate and the dividend lost. On the other hand, an Asian-style futures contract is settled against the arithmetic average of the underlying asset price quotations taken over the settlement period. Due to the Asian-style settlement procedure, the fair value of the Asian-style futures contract during the settlement period is different from the plain vanilla futures contract.

Theorem 1. *The fair value of an Asian-style futures contract at time t, denoted by V_t, is given by*

$$V_t = \frac{S_{t_1} + S_{t_2} + ... + S_{t_n} + (N-n)S_t}{N}, \tag{1}$$

where $t_n \leq t < t_{n+1}$, S_t is the underlying asset price at time t, t_n is the n-th time mark where the n-th underlying asset price quotation is taken and N denotes the total number of time marks.

Proof. Assume the effect of interest rate within the settlement period is negligible and the market is frictionless. At any time t in the settlement period, suppose one buys an Asian-style futures contract and prepare an amount of cash equals to the exercise price of the contract, where the cash will later be used to settle the contract at maturity. On the other hand, one purchases the underlying asset (for index futures, one purchases a whole basket of underlying index constituent stocks, along with their respective weights.) with value equals to [(contract multiplier) $\times \frac{(N-n)S_t}{N}$] and hold an amount of cash with value equals to [(contract multiplier) $\times \frac{S_{t_1}+S_{t_2}+...+S_{t_n}}{N}$]. At each time mark t_i, where $i = n+1, n+2, ..., N$, one unwinds the underlying asset position by selling an amount equals to $\frac{S_{t_i}}{N}$. The values of both portfolios will be equal to the settlement price at maturity and hence one can conclude that both portfolios have equal value at all time before maturity. □

Taking partial differentiation on both sides of (1) with respect to S_t, one obtains the delta of the Asian-style futures at time t:

$$\frac{\partial V_t}{\partial S_t} = \frac{N-n}{N}, \text{ where } t_n \leq t < t_{n+1}. \tag{2}$$

Note that the delta decreases by $\frac{1}{N}$ when a time mark is passed. Thus arbitrageurs need to unwind a part of the offsetting position at each time mark in order to track the decreasing delta of the Asian-style futures.

In actual circumstances, a futures contract exhibits no arbitrage opportunity if the following condition holds:

$$V_t - T_t \leq F_t^{ask} \quad and \quad F_t^{bid} \leq V_t + T_t, \tag{3}$$

where V_t is the fair value of the futures contract at time t from (1), F_t^{bid} and F_t^{ask} are the bid and ask price of the futures contract at time t respectively and T_t is the round-trip transaction costs needed to construct and maintain an arbitrage portfolio for each index futures contract at time t.

If the lower boundary of (3) is violated (i.e. $V_t - T_t > F_t^{ask}$), then an underpricing is identified and the magnitude of mispricing is given by $[V_t - T_t] - F_t^{ask}$. On the other hand, if the upper boundary of (3) is violated (i.e. $F_t^{bid} > FV_t + T_t$), then an overpricing is identified and the magnitude of mispricing is given by $F_t^{bid} - [FV_t + T_t]$. If (3) holds, then no mispricing is identified and the magnitude of mispricing is given by 0. In the remaining part of this section, we will examine the pattern of mispricing of the HSI Futures on the expiration days.

3 Trading Strategy

In this section we introduce a practical trading strategy which is an integer program to exploit the mispricing opportunity using a proxy of the underlying asset. In general, this trading strategy can be applied to any Asian-style futures arbitrage during the settlement period. The integer program can detect mispricing, construct an arbitrage portfolio and dynamically maintain the arbitrage portfolio.

3.1 Integer Program

At time t on the expiration day, where $t_n \leq t < t_{n+1}$, the integer program is formulated as:

$$\text{Maximize} \quad Q_t = (V_t - F_t^{ask} - T_t)K_{1,t}^+ + (V_t - F_t^{bid} + T_t)K_{1,t}^-, \tag{4a}$$

$$\text{Subject to:} \quad -\epsilon_t \leq \frac{(N-n)}{N} \cdot (K_{1,t} + K_{1,t}^+ + K_{1,t}^-) + \beta_t \cdot K_{2,t} \leq \epsilon_t, \tag{4b}$$

$$-\epsilon_{t_i} \leq \frac{(N-i)}{N} \cdot (K_{1,t} + K_{1,t}^+ + K_{1,t}^-) + \beta_{t_i} \cdot K_{2,t_i} \leq \epsilon_{t_i}, \qquad i = n+1, n+2, ..., N, \tag{4c}$$

$$0 \leq K_{1,t}^+ \leq M_t^+, -M_t^- \leq K_{1,t}^- \leq 0, \tag{4d}$$

$$-C \leq K_{1,t} + K_{1,t}^+ + K_{1,t}^- \leq C, \tag{4e}$$

where $K_{1,t}$ is the position on the futures at time t, $K_{1,t}^+$ and $K_{1,t}^-$ are quantities of the futures to buy and sell at time t respectively, F_t^{ask} and F_t^{bid} are the ask and bid prices of the futures contract at time t respectively, β_t is the delta of the proxy at time t, $K_{2,t}$ is the position on the proxy at time t, ϵ_t is an endurance level for the delta of the arbitrage portfolio at time t, M_t^+ and M_t^- are the total sizes of bid and ask quotes in the market at time t respectively, C is the investor's capital control, T_t is the round-trip transaction cost needed to construct and maintain the arbitrage portfolio for each futures contract at time t. Note that $K_{1,t}^+$ is non-negative and $K_{1,t}^-$ is non-positive. In particular, if $K_{1,t}^+ = A$ then the investor is buying A units of the futures contract, if $K_{1,t}^- = -A$ then the investor is selling A units of the futures contract.

The optimal solution of the integer program gives us an initial position on the futures contract and the proxy ($K_{1,t}^+$, $K_{1,t}^-$ and $K_{2,t}$) and the amount of proxy we need to hold at any time (K_{2,t_i}, for $i = n+1, ..., N$) in order to maintain the delta of the arbitrage portfolio. Next we discuss in detail the integer program in the remaining part of Section 3.

3.2 Explanation of the Objective Function

The objective function of the integer program (4a) is used to detect mispricing and maximize the payoff once a mispricing occurs. Proposition 1 shows that the integer program will not generate a buy singal and a sell signal of the futures contracts at the same time.

Proposition 1. *The optimal* $K^+_{1,t}$ *and* $K^-_{1,t}$ *cannot be both non-zero.*

Proof. Suppose in contrary that the optimal $K^+_{1,t}$ and $K^-_{1,t}$ are both non-zero. Because of the constraints of the integer program (4d), $K^+_{1,t}$ is positive and $K^-_{1,t}$ is negative. We let the optimal $K^+_{1,t} = a > 0$ and $K^-_{1,t} = b < 0$. Note that $F^{ask}_t > F^{bid}_t$, we divide our proof into three cases. Case (i): $V_t > F^{ask}_t > F^{bid}_t$. We have $V_t - F^{ask}_t > 0$ and $V_t - F^{bid}_t > 0$, but a feasible solution $K^+_{1,t} = a - b$ and $K^-_{1,t} = 0$ yields an objective value that is greater than the optimal value and it is a contradiction. Case (ii): $F^{ask}_t > V_t > F^{bid}_t$. We have $V_t - F^{ask}_t < 0$ and $V_t - F^{bid}_t > 0$, but a feasible solution $K^+_{1,t} = K^-_{1,t} = 0$ yields an objective value that is greater than the optimal value and it is also a contraction. Case (iii): $F^{ask}_t > F^{bid}_t > V_t$. The proof is similar to case (i). □

Moreover, when (3) is violated, i.e. $F^{ask}_t < V_t - T_t$ or $F^{bid}_t > V_t + T_t$, the objective value of the integer program becomes positive and that indicates a mispricing. In fact, the optimal value of the integer program is how much one can gain from an arbitrage opportunity if one can construct a perfectly hedged arbitrage portfolio, i.e. one can trade the underlying asset.

3.3 Explanation of the Constraints

Firstly for (4b) and (4c), the middle expressions are the delta of the arbitrage portfolio at time t and the future time marks t_i (for $i = n + 1, n + 2, ..., N$) respectively and their magnitudes are bounded by ϵ_{t_i}. Hence (4b) and (4c) are used to restrict the delta of the arbitrage portfolio under the endurance level of the investor. Since we use the proxy instead of the underlying asset to perform arbitrage, we need to keep the magnitudes of the delta below the endurance level. Note that the delta of the index futures contract will not change between two consecutive time marks, so it is sufficient to restrict the delta at each time mark.

Secondly, (4d) is used to keep the trading quantities of the futures contract from exceeding the quantities that the market can offer.

Thirdly, (4e) is used to keep the position on the futures from exceeding a capital control set by the investor. Since the position on the futures and the delta of the arbitrage portfolio are bounded, the position on the proxy is bounded as well. Hence by setting the capital control on the futures contracts, the position of both futures and proxy are restricted from exceeding the investor's capital.

3.4 Maintaining the Arbitrage Portfolio

In (2) one can see that the delta of the Asian-style futures is decreasing during the settlement period, so we need to keep unwinding the position on the proxy in order to maintain the arbitrage portfolio. In fact the integer program can also be used to maintain the arbitrage portfolio. The output K_{2,t_i} for $i = n+1, n+2, ..., N$ gives the investor the position of the proxy they should hold in different time. In other words, the value of $(K_{2,t_i} - K_{2,t_{i-1}})$ tells the investor how much proxy they should unwind at future time marks t_i for $i = n + 1, n + 2, ..., N$.

Moreover, if one already has an arbitrage portfolio and the integer program spots a mispricing, then a new arbitrage portfolio will be constructed. The integer program will output a set of new $K_{2,t}$'s and the old $K_{2,t}$'s will be updated. So the investor can maintain the new arbitrage portfolio according to the new output of the integer program.

4 Non-profitable Mispricing Opportunity and Solution

In the last section, we introduce the trading strategy to exploit mispricing opportunities. However, we have not considered some factors that can erode the profitability.

In Section 4.1 we address two factors when implementing the strategy in practice, namely that the arbitrage portfolio is not delta neutral and the delta of the proxy may deviate after the arbitrage portfolio is set up. Both of the factors may erode the profitability or even cause a loss. In Section 4.2, we introduce an approach to filter out the non-profitable mispricing opportunities. In particular, the case of using next expiring futures contracts as a proxy to perform arbitrage of expiring futures contracts is used as an illustration.

4.1 Factors Eroding the Profitability

The optimal value of the integer program is equal to the amount an arbitrageur can receive using the underlying asset to perform arbitrage. However this strategy employs the proxy to construct the portfolio, the actual payoff the investor can receive is differ from the optimal value of the integer program, e.g. in some cases the optimal value is positive but the actual payoff is negative. In view of this, Theorem 2 gives a condition on the optimal value such that the trading strategy is guaranteed to be profitable. For simplicity of notation, we only show the case of $t = t_n$, while the general case $t_n \leq t < t_{n+1}$ follows easily.

Theorem 2. *The trading strategy is guaranteed to be profitable at time t_n if the optimal value of the integer* *program is greater than a threshold δ_{t_n}, where δ_{t_n} is given by the following:*

$$\delta_{t_n} = \sum_{i=n+1}^{N} [(K_{2,t_i} - K_{2,t_{i-1}}) \cdot \alpha_{t_i} - \frac{K_{1,t}^+ + K_{1,t}^-}{N}] \cdot \Delta S_{t_i} + \sum_{i=n+1}^{N} (K_{2,t_i} - K_{2,t_{i-1}})(S_{t_n} \cdot \Delta\alpha_{t_i}), \tag{5}$$

where S_t is the price of the underlying asset at time t, $\Delta S_t = S_t - S_{t_n}$, $\alpha_t = \frac{P_t}{S_t}$ and $\Delta\alpha_t = \alpha_t - \alpha_{t_n}$. This expression is valid for both long and short arbitrage portfolios.

Proof. Consider an arbitrage portfolio set up at time t_n using a proxy, the actual payoff of an arbitrage opportunity A_{t_n} is equal to the profit/loss of the futures contracts plus the profit/loss of the proxy minus the round trip transaction costs. Hence A_{t_n} can be expressed as:

$$A_{t_n} = (K_{1,t}^+ + K_{1,t}^-)(F_{t_N} - F_{t_n}) - \sum_{i=n+1}^{N} (K_{2,t_i} - K_{2,t_{i-1}})(P_{t_i} - P_{t_n}) - (K_{1,t}^+ - K_{1,t}^-) \cdot T_{t_n},$$

where F_t is the trading price of the futures contracts at time t and P_t is the trading price of the proxy at time t. On the other hand, the arbitrage portfolio is perfectly hedged if one can trade the underlying asset. Thus the optimal value of the integer program Q_{t_n} is equal to the profit/loss of the futures contracts plus the profit/loss of the underlying asset minus the round trip transaction costs. Therefore Q_{t_n} can be expressed as:

$$Q_{t_n} = (K_{1,t}^+ + K_{1,t}^-)(F_{t_N} - F_{t_n}) - \sum_{i=n+1}^{N} \frac{K_{1,t}^+ + K_{1,t}^-}{N}(S_{t_i} - S_{t_n}) - (K_{1,t}^+ - K_{1,t}^-) \cdot T_{t_n}.$$

If $A_{t_n} > 0$, the mispricing opportunity is profitable. In view of that, we define the threshold as the optimal value minus the actual payoff and we require the optimal value of the integer program to be greater than the threshold, i.e. $Q_{t_n} > Q_{t_n} - A_{t_n} := \delta_{t_n}$. By subtracting A_{t_n} from Q_{t_n}, we get (5). □

When $\delta_t < 0$, an arbitrageur gains more than the optimal value of the integer program. Conversely, if $\delta_t > 0$, the arbitrageur gains less than the optimal value. Moreover if $\delta_t > Q_t > 0$, the optimal value is positive but the mispricing opportunity is not profitable. For the case of using the first back month futures as the proxy to perform arbitrage on the front month futures, (5) suggests two factors causing the actual payoff to be different from the optimal value: firstly, the arbitrage portfolio is not delta neutral; secondly, the "delta" ($\alpha_t = \frac{P_t}{S_t}$) of the proxy (the ratio of the price of the first back month futures to the price of the underlying asset) may deviate after the arbitrage portfolio is set up. These two factors correspond to the first term and second term of (5) respectively and they are caused by using a proxy instead of the underlying asset.

For the first factor, as the arbitrage portfolio is not delta neutral, any price movement of the underlying asset causes the value of the arbitrage portfolio to change. If the price of the underlying asset moves in an unfavorable direction, the value of the arbitrage portfolio will drop. For the second factor, the fair value of a first back month futures contract is given by the current price of the underlying asset multiplying the interest rate and dividend lose. Hence theoretically the delta of the first back month futures contract is given by the price of the first back month contract divided by the price of the underlying asset. In reality the price of the futures contract may include noise, hence we call the ratio $\alpha_t = \frac{P_t}{S_t}$ "delta" of the first back month futures to distinguish it from the theoretical delta. Moreover $\triangle\alpha_t$ is the change of the "delta". Due to factors such as order imbalance, investors' optimism/pessimism and reaction to market news etc., the "delta" of the first back month contracts may deviate after the portfolio is constructed. Moreover if the "delta" moves in an unfavorable direction (e.g. one has already set up a long/short arbitrage portfolio but the "delta" of the proxy increases/decreases), the value of the arbitrage portfolio will also drop.

4.2 Filtering out Non-profitable Mispricing Opportunity

When $Q_t > \delta_t$, Theorem 2 guarantees that the trading strategy is profitable. However, one needs to know future prices $S_{t_i}, i = n+1, n+2, ..., N$, in order to calculate the threshold δ_t and it is impossible to know them in real practice. In view of that, we conservatively estimate the threshold δ_t and require the optimal value of the integer program to be greater than an estimated threshold in order to filter out non-profitable mispricing opportunities.

In this paper, we conservatively estimate the threshold δ_t by $\hat{\delta}_t$ from past data. Again for simplicity of notation, we only give the expression for the case $t = t_n$, while the general case $t_n \leq t < t_{n+1}$ follows easily. At time t_n, the expression of $\hat{\delta}_{t_n}$ is given as follows:

$$\begin{aligned}\hat{\delta}_{t_n} &= \sum_{i=n+1}^{N} |[(K_{2,t_i} - K_{2,t_{i-1}}) \cdot \hat{\alpha}_t - \frac{K_{1,t}^{+} + K_{1,t}^{-}}{N}] \cdot \triangle \hat{S}_t| + \sum_{i=n+1}^{N} |(K_{2,t_i} - K_{2,t_{i-1}})(S_{t_n} \cdot \Delta\hat{\alpha}_t)| \\ &= \sum_{i=n+1}^{N} |[(K_{2,t_i} - K_{2,t_{i-1}}) \cdot \hat{\alpha}_t - \frac{K_{1,t}^{+} + K_{1,t}^{-}}{N}] \cdot \triangle \hat{S}_t| + |(K_{2,t_n})(S_{t_n} \cdot \Delta\hat{\alpha}_t)|.\end{aligned} \tag{6}$$

For α_{t_i}, $\triangle\alpha_{t_i}$ and $\triangle S_{t_i}$ in (5), where $i = n+1, n+2, ..., N$, we use $\hat{\alpha}_t$ $\triangle\hat{\alpha}_t$ and $\triangle\hat{S}_t$ as their estimations respectively. In particular, $\hat{\alpha}_t$ is used as an estimation for all α_{t_i}, $i = n+1, n+2, ..., N$ and so are the other two estimations.

We first collect the "delta" of the next expiring futures at each passed time mark to form a set of data $\{\alpha_{t_i}\}_{i=1}^{n}$. We assume that the "delta" at different time marks are normally distributed. We estimate the delta of the next expiring futures by using the mean of the data $\{\alpha_{t_i}\}_{i=1}^{n}$, i.e. $\hat{\alpha}_t = \sum_{i=1}^{n} \alpha_{t_i}/n$, where $t_n \leq t < t_{n+1}$. In this paper we also input $\hat{\alpha}_t$ as β_t (the delta of the proxy) in the integer program.

Moreover, we consider a prediction interval of α_t at a confidence level θ which is to be chosen by the investor in practice. The prediction interval is given by $\hat{\alpha}_t \pm t_{1-\theta/2,n-1}s_t(1+\frac{1}{n})^{\frac{1}{2}}$, where $t_{1-\theta/2,n-1}$ is the t-value at the corresponding confidence level and degree of freedom and s_t is the sample standard deviation of the data $\{\alpha_{t_i}\}_{i=1}^{n}$. For detail of prediction interval, see Hahn (1970). For a short arbitrage opportunity, i.e. we want to set up a long position on the next expiring futures, it will cause a gain if the "delta" increases and a loss if the "delta" decreases. So $\triangle\hat{\alpha}_t$ is set as $\alpha_t - \hat{\alpha}_t + t_{1-\theta/2,n-1}s_t(1+\frac{1}{n})^{\frac{1}{2}}$, i.e. how much the "delta" can possibly decrease at a level of confidence θ. Similarly, for a long arbitrage opportunity, $\triangle\hat{\alpha}_t$ is set as $\hat{\alpha}_t + t_{1-\theta/2,n-1}s_t(1+\frac{1}{n})^{\frac{1}{2}} - \alpha_t$ to estimate the increase of the "delta". Finally $\triangle\hat{S}_t$ is set as the value at risk (VaR) of the underlying asset over the last η days at a confidence level ζ which is to be chosen by the investor in practice. More specifically, the way to calculating VaR is that daily high and low price data of the underlying asset over the last η days is collected and we calculate the difference between daily high and low price for each trading day. We then take the ζ-th greatest change between daily high and low price as the VaR.

5 Process of Implementing the Strategy

In this section we summarize the process on how to implement our strategy by using the proxy. It is given as pseudocode in Algorithm 1.

Algorithm 1 Process of implementing the strategy

Choose a proxy
Define an estimation $\hat{\delta}_t$ for the threshold δ_t
Choose values of the parameters C and ϵ_t of the integer program
while the futures contracts is not yet matured **do**
    Solve the integer program (4)
    if $Q_t > \hat{\delta}_t$ **then**
        Set up an arbitrage portfolio
    else if $t =$ time marks **then**
        Maintain the arbitrage portfolio based on the output $K_{2,t}$'s of the integer program
    end if
end while

6 Data Used for Empirical Test

In this paper, Hang Seng Index Futures (HSI Futures) of the Hong Kong market will be used to examine mispricing patterns on the expiration day and test our trading strategy. The HSI Futures was introduced in May, 1986. Its underlying asset is Hang Seng Index (HSI), a capitalization-weighted index consisting of 50 major stocks of the Hong Kong market. The futures contracts are traded in Hong Kong Exchanges and

Clearing Limited, and expire on the day before the last business day of every month. It has a contract multiplier of 50 Hong Kong dollars per index point. An Asian-style settlement procedure is used to calculate the settlement price of the HSI Futures, which is the arithmetic average of the HSI quotations taken every five minutes on the expiration day, rounding down to the nearest whole number. The majority of trading volumes of the HSI Futures contracts are concentrated in front month contracts and first back month contracts.

Our data consists of every two seconds quotations of the HSI and tick-by-tick bid/ask quote records of the HSI Futures on the expiration days in the period from Jan 2013 through June 2015, which provides in total 30 expiration days to study. More specifically, it is a historical intraday data recording the price of the HSI and the HSI Futures on the expiration days from 9:35 a.m. (first HSI quotation is taken) to 4:00 p.m. (market closing).

7 Pattern of Mispricing of HSI Futures

In this section we examine patterns of mispricing of the Hang Seng Index Futures (HSI Futures) on the expiration days. In Sections 7.1 and 7.2, we examine the frequency and the magnitude of mispricing of the HSI Futures on the expiration days respectively.

7.1 Frequency of Mispricing

Transaction cost is an important factor to be considered before performing arbitrage, as high transaction costs may completely erode the profitability. In view of this, we examine the frequency of mispricing at different levels of transaction costs. Table 2 reports the frequency of mispricing signals at four levels of transaction costs: 0, 1 index point, 0.0077% and 0.1077% for a total of $9,750$ observations. In particular, these 4 levels correspond to no transaction cost, brokerage's rates of transaction costs for trading index futures, ETFs and stocks in the Hong Kong market respectively. It is worth noting that since we consider bid-ask quotes of the index futures instead of trade quotes, it is possible to generate a neutral signal even with 0 transaction cost (no mispricing occurs when (3) holds). In fact, 18.5% of the observations are identified as no mispricing with 0 transaction cost. Almost 60% and about half of the observations are identified as mispricing when the transaction costs are 1 index point and 0.0077% respectively. When transaction costs increase to the level of trading stocks, the proportion of mispricing drastically drops. Only 0.2% of the observations are recorded as mispricing at 0.1077% transaction costs, implying arbitrage opportunities using constitute stocks are scarce even for brokers. This observation is similar to the finding of Modest and Sundaresan (1983) in the U.S. market, where the actual futures prices fluctuate within a bounded interval without showing any arbitrage opportunity when trading index constituent stocks.

Although there is only a few mispricing opportunities at the rate of the transaction costs of trading index constituent stocks, the opportunities with trading index futures and ETFs are abundant due to relatively low transaction costs. This shows the potential of ETFs and the first back month futures being used as a proxy to construct the arbitrage portfolio.

No significant difference between the frequency of underpricing and overpricing is observed at 0, 1 index point and 0.0077% rate of transaction costs. The binomial test however indicates that the frequency of underpricing is significantly greater than the frequency of overpricing at 0.1077% rate of transaction costs with a p-value of 0.0013. This shows consistency with current "Regulated Short Selling" in the Hong Kong market, where short sale of stocks could not be made below the current best ask price.

Figure 1 shows the rate of break-even transaction costs versus time. The upper dashed and lower solid horizontal lines represent the rate of transaction costs of trading stocks and ETFs for brokerage firm respectively. Each curve represents the rate of break-even transaction costs at different time on one expiration day and there are a total of 30 curves in the figure to represent 30 expiration days. We note that no substantial increasing or decreasing trend of the break-even transaction costs is observed over the whole period in the expiration days.

Rate of transaction costs	Total obs.	Underpricing	No mispricing	Overpricing
0	9750	3875	1808	4067
		39.7%	18.5%	41.7%
1 index point	9750	2780	3917	3053
(Brokerage's rate for index futures)		28.5%	40.2%	31.3%
0.0077%	9750	2203	5118	2429
(Brokerage's rate for ETFs)		22.6%	52.5%	24.9%
0.1077%	9750	17	9730	3
(Brokerage's rate for stocks)		0.17%	99.8%	0.03%

Table 2: Frequency of mispricing

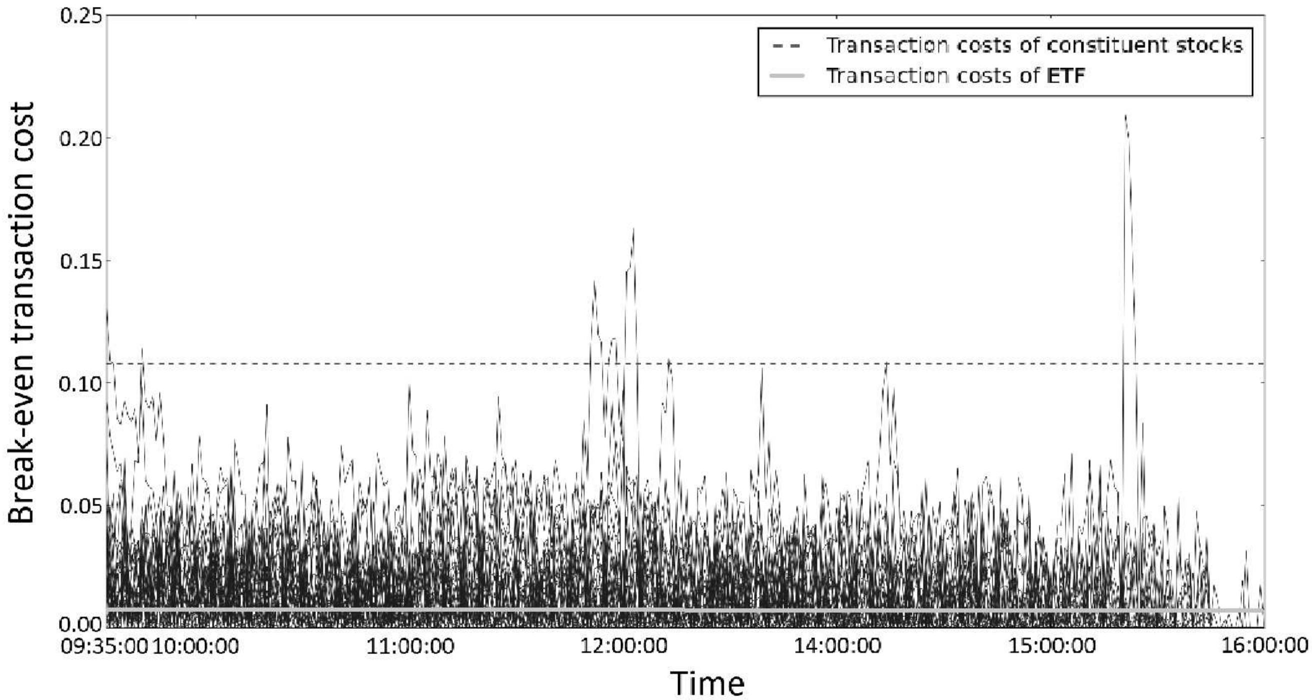

Figure 1: Break-even transaction costs over time

7.2 Magnitude of Mispricing

Figure 2 shows the magnitude of mispricing at 0 transaction cost versus time for the 30 expiration days. Each curve represents magnitude of mispricing over time on one expiration day and there are 30 curves to represent 30 expiration days. A trend of decreasing magnitudes of mispricing with closer time to maturity can be observed. In the former-part of the expiration days the magnitudes of mispricing are relatively greater, some unusually large mispricings are also spotted. In the latter-part of the expiration days, the mispricings are converging to zero and they eventually become zero at the market close. This observation is consistent with findings of MacKinlay and Ramaswamy (1988) and Yadav and Pope (1994), implying the existence of factors influenced by time to expiration. Possible factors include risk of offsetting the index futures with only a proxy of a whole basket of constituent stocks, which is greater with longer time to maturity. Since price movements of the underlying index is not perfectly duplicated, arbitrageurs may bear potential loss at each five-minute mark when unwinding the offset position so they have to look for a greater margin of deviation with longer time to maturity. Another factor could be transaction costs involved in trading proxy. Because of the decreasing delta of the futures contract, a smaller amount of proxy is needed to construct an arbitrage portfolio with closer time to maturity. Hence fewer transaction costs are needed, arbitrageurs can then profit from a smaller size of mispricing with closer time to maturity.

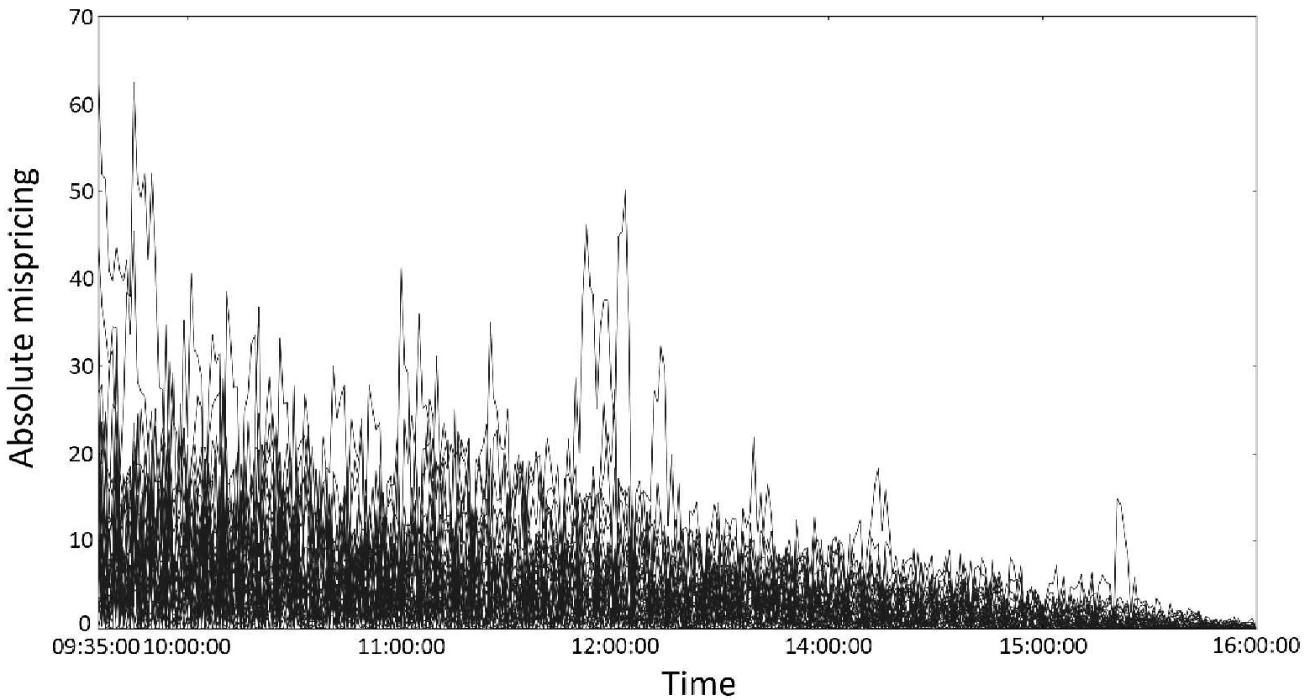

Figure 2: Magnitude of mispricing over time

8 Empirical Results

As we discussed in Section 7.1, the arbitrage opportunities on the expiration days are scarce using constituent stocks due to costly transaction costs of stock trading in the Hong Kong market, there are, however other assets featuring low transaction costs available in the market, e.g. ETFs and the first back month futures contract. Moreover, there are a decent amount of mispricing using these assets. This shows a potential of using these assets as a proxy to perform arbitrage.

In this section, we report the empirical results of applying our trading strategy discussed in Section 3 and Section 4 to back-test the historical data mentioned in Section 6, i.e. intraday data of HSI Futures on the expiration days from January 2013 through June 2015. In particular, we look for arbitrage opportunities of the front month contracts and use the first back month contracts as the proxy. Tables 3 to 6 summarize the profit and the number of trades on each expiration day by applying our trading strategy with 90%, 95%, 99% and 99.9% confidence level respectively where the past $\eta = 250$ days historical daily HSI price data is used to calculate the VaR. More specifically, a 90% confidence level means that we take both θ and ξ equal to 90% in calculating $\hat{\delta}_t$.

For the parameters in the integer program, there are $N = 66$ time marks for the HSI Futures on the expiration day. We consider an arbitrage portfolio consisting of 66 front month index futures contracts as 1 set of arbitrage portfolio. Also we set the capital control $C = 66 \times 2 = 132$, i.e. one can at most possess 2 sets of arbitrage portfolio at the same time. In addition, we put one more restriction: we can only set up one set of arbitrage portfolio at any one time. We set the endurance level $\epsilon_t = 0.05$ at any time t for the delta of the arbitrage portfolio. We set up an arbitrage portfolio if the optimal value of the integer program Q_t is positive and greater than the estimated threshold $\hat{\delta}_t$. In this empirical test, the round-trip transaction costs for each index futures contract is set to be 1 index point, which is the level of brokerage transaction costs. We implement the trading strategy at every whole minute. The front month and the first back month HSI Futures contracts are usually liquid enough such that one can set up at least one set of arbitrage portfolio. Note that if one possesses a long arbitrage portfolio and are setting up a short arbitrage portfolio, the two positions will be offset by each other and vice versa.

By using our trading strategy, one can generate positive total profit among all 4 confidence levels. The total profit with 90%, 95%, 99% and 99.9% confidence levels are 27647, 26265, 10430 and 51 index points respectively. This shows a risk premium relationship between the total return and the confidence level — the more risk you are willing to bear, the more return you can generate. Moreover, if one increases the confidence level, the estimation of the threshold will increase. Hence more mispricing opportunities will be filtered out and there are less number of trades. When the confidence level increases to 99%, all trades with negative return are filtered out and all trades can yield a positive return. Suppose the deposit required for trading each index futures contract is 3000 index points (a conservative estimation of deposit for the Hong Kong market), one needs $792,000$ index points as an initial capital since the capital control for front month contracts is $C = 66 \times 2 = 132$ in the integer program and we need the same amount of first back month contracts. For such initial capital and for the case of 90%, 95%, 99% and 99.9% confidence level, our strategy can generate an average daily return of 0.12%, 0.11%, 0.04% and 0.0002% respectively.

The trading strategy introduced in this paper is simple to implement—it is based on an integer programming technique and there are many libraries available for integer programming. Moreover our approach requires only little market information: daily data of the underlying index and intraday data of first back month futures and underlying index on the expiration day. Both data are easily accessible and even "small investors" who lack of market information can use our strategy.

Expiration day	Profit(index point)	No. of trades
2013-01-30	0	0
2013-02-27	601	1
2013-03-27	0	0
2013-04-29	0	0
2013-05-30	0	0
2013-06-27	0	0
2013-07-30	0	0
2013-08-29	0	0
2013-09-27	0	0
2013-10-30	0	0
2013-11-28	0	0
2013-12-30	0	0
2014-01-29	0	0
2014-02-27	0	0
2014-03-28	0	0
2014-04-29	0	0
2014-05-29	0	0
2014-06-27	0	0
2014-07-30	0	0
2014-08-28	0	0
2014-09-29	0	0
2014-10-30	-439	2
2014-11-27	662	1
2014-12-30	-508	2
2015-01-29	0	0
2015-02-26	0	0
2015-03-30	756	1
2015-04-29	4276	2
2015-05-28	9490	5
2015-06-29	12809	2
Total	27647	16

Table 3: Empirical result with 90% confidence level

Expiration day	Profit(index point)	No. of trades
2013-01-30	0	0
2013-02-27	0	0
2013-03-27	0	0
2013-04-29	0	0
2013-05-30	0	0
2013-06-27	0	0
2013-07-30	0	0
2013-08-29	0	0
2013-09-27	0	0
2013-10-30	0	0
2013-11-28	0	0
2013-12-30	0	0
2014-01-29	0	0
2014-02-27	0	0
2014-03-28	0	0
2014-04-29	0	0
2014-05-29	0	0
2014-06-27	0	0
2014-07-30	0	0
2014-08-28	0	0
2014-09-29	0	0
2014-10-30	-464	2
2014-11-27	662	1
2014-12-30	-508	2
2015-01-29	0	0
2015-02-26	0	0
2015-03-30	0	0
2015-04-29	4276	2
2015-05-28	9490	5
2015-06-29	12809	2
Total	26265	14

Table 4: Empirical result with 95% confidence level

Expiration day	Profit(index point)	No. of trades
2013-01-30	0	0
2013-02-27	0	0
2013-03-27	0	0
2013-04-29	0	0
2013-05-30	0	0
2013-06-27	0	0
2013-07-30	0	0
2013-08-29	0	0
2013-09-27	0	0
2013-10-30	0	0
2013-11-28	0	0
2013-12-30	0	0
2014-01-29	0	0
2014-02-27	0	0
2014-03-28	0	0
2014-04-29	0	0
2014-05-29	0	0
2014-06-27	0	0
2014-07-30	0	0
2014-08-28	0	0
2014-09-29	0	0
2014-10-30	0	0
2014-11-27	0	0
2014-12-30	51	1
2015-01-29	0	0
2015-02-26	0	0
2015-03-30	0	0
2015-04-29	2725	1
2015-05-28	7564	3
2015-06-29	0	0
Total	10430	5

Table 5: Empirical result with 99% confidence level

Expiration day	Profit(index point)	No. of trades
2013-01-30	0	0
2013-02-27	0	0
2013-03-27	0	0
2013-04-29	0	0
2013-05-30	0	0
2013-06-27	0	0
2013-07-30	0	0
2013-08-29	0	0
2013-09-27	0	0
2013-10-30	0	0
2013-11-28	0	0
2013-12-30	0	0
2014-01-29	0	0
2014-02-27	0	0
2014-03-28	0	0
2014-04-29	0	0
2014-05-29	0	0
2014-06-27	0	0
2014-07-30	0	0
2014-08-28	0	0
2014-09-29	0	0
2014-10-30	0	0
2014-11-27	0	0
2014-12-30	51	1
2015-01-29	0	0
2015-02-26	0	0
2015-03-30	0	0
2015-04-29	0	0
2015-05-28	0	0
2015-06-29	0	0
Total	51	1

Table 6: Empirical result with 99.9% confidence level

9 Conclusion

In this paper, we propose a practical trading strategy for Asian-style futures arbitrage. In particular, our trading strategy is an integer program which uses a proxy to construct the arbitrage portfolio. Since the arbitrage portfolio is not delta neutral and the delta of the proxy may vary after the portfolio is set up, we introduce a threshold to filter out some non-profitable mispricing opportunities. HSI Futures of the Hong Kong market is used to test our trading strategy. We first examine the pattern of mispricing of the HSI Futures on the expiration days. We show that there is a trend of decreasing magnitude of mispricing when time approaches maturity. Moreover, we find that there is a decent amount of frequency of mispricing, showing some potential arbitrage opportunities. Empirical results show that one can generate trades that are profitable using a conservative estimation of the threshold. Our strategy is general enough not only to be applied to the HSI Futures, but also to any futures with an Asian-style settlement procedure.

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