THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics

MATH 2050B Mathematical Analysis I

Tutorial 10 (November 14)

The following were discussed in the tutorial this week:

1. Definition of uniformly continuous function, nonuniform continuity criteria, uniform continuity theorem,

2. Determine if the following functions are uniformly continuous:

   (a) $f : [0, \infty) \to \mathbb{R}$ defined by $f(x) = \sqrt{x}$,

   (b) $f : \mathbb{R} \to \mathbb{R}$ definedy by $f(x) = \cos(x^2)$.

3. Prove that if f is uniformly continuous on a bounded subset A of $\mathbb{R}$, then f is bounded on A.

4. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function. Suppose $\lim_{x\to-\infty} f(x) = \lim_{x\to\infty} f(x) = \ell \in \mathbb{R}$. Show that f is uniformly continuous on $\mathbb{R}$.

5. A function $f : \mathbb{R} \to \mathbb{R}$ is said to be periodic on $\mathbb{R}$ if there exists a number $p > 0$ such that $f(x + p) = f(x)$ for all $x \in \mathbb{R}$. Prove that a continuous periodic function on $\mathbb{R}$ is uniformly continuous on $\mathbb{R}$.