HW 2 Due Feb 16, 2017

1. Prove Lagrange's identity:

$$\left|\sum_{i=1}^{n} a_i b_i\right|^2 = \left(\sum_{i=1}^{n} |a_i|^2\right)\left(\sum_{i=1}^{n} |b_i|^2\right) - \sum_{1\le i<j\le n} |a_i \bar{b}_j - a_j \bar{b}_i|^2$$

for complex numbers a_i, b_i, $i = 1, 2, \cdots, n$.

2. Compute $\int_{|z|=r} y\,dz$, where $y = \operatorname{Im} z$, in the counterclockwise direction.

3. Compute $\int_{|z|=1} z^m \bar{z}^n\,dz$ in the counterclockwise direction for any integers m and n.

4. Show that for $R > 2$,

$$\left|\int_{|z|=R} \frac{3z-1}{z^4+4z^2+3}\,dz\right| \le \frac{2\pi R(3R+1)}{(R^2-1)(R^2-3)}.$$

5. Show that $\left|\int_{\gamma_R} \frac{2e^z}{1+e^{3z}}\,dz\right| \le \frac{8\pi e^R}{e^{3R}-1}$

where γ_R is the vertical line segment from R to $R+4\pi i$ for $R > 0$.

6. (a) Is it possible to find an antiderivative of the function $f(z) = \frac{1}{z}$ on $\mathbb{C} \setminus \{0\}$?

(b) Same question as in (a) with $f(z)$ replaced by $g(z) = \frac{1}{z^2}$.

7. Suppose that $f(z)$ is analytic on a domain Ω which is symmetric with respect to the real axis. Show that $g(z) = \overline{f(\bar{z})}$ is a well defined analytic function on Ω.