MATH3060 HW7 Due date: Nov 26, 2021 (at 12:00 noon)

1. Show that if E is nowhere dense in a metric space X, then
   (a) the closure $\overline{E}$ in X is also nowhere dense in X.
   (b) every subset of E is also nowhere dense in X.

2. Show that in $C[0,1]$,
   (a) $\{f \in C[0,1] : \int_0^1 f(x)dx \neq 0\}$ is dense;
   (b) $\{f \in C[0,1] : f(0.1) = 2\}$ is nowhere dense.

3. Let $l_2 = \{\{x_n\}_{n=1}^{\infty} : \sum_{n=1}^{\infty} x_n^2 < \infty\}$ with metric

$$d_2(\{x_n\},\{y_n\}) = \sqrt{\sum_{n=1}^{\infty}(x_n-y_n)^2}.$$

   Show that $H = \{\{x_n\}_{n=1}^{\infty} \in l_2 : |x_n| \leq \frac{1}{n}\}$ is nowhere dense in (l_2, d_2).

4. Show that the boundary of a nonempty open set in a metric space must be closed and nowhere dense. Conversely, every closed nowhere dense set is the boundary of some open set.

(End)