MATH3060 HW1 Due date: Sep 24, 2021 (at 12:00 noon)

1. Each of the following functions are defined on $(-\pi, \pi]$. Sketch the 2π-periodic extension, find the corresponding Fourier expansion, and discuss the pointwise convergence (using Thm 1.5 and 1.6)

(a) $f_1(x) = \begin{cases} x, & x \in [0,\pi] \\ -x, & x \in (-\pi, 0) \end{cases}$

(b) $f_2(x) = \begin{cases} 1, & x \in [0,\pi] \\ 0, & x \in (-\pi, 0) \end{cases}$

(c) $f_3(x) = e^{-2x}$

(2) Show that the function

$$f(x) = \begin{cases} |x|^{1/2} \sin\frac{1}{x}, & x \in [-\pi, \pi] \setminus \{0\} \\ 0, & x = 0 \end{cases}$$

is not Lipschitz continuous at $x = 0$.

(3) Consider the function $f(x) = x^2$ on $(0, 2\pi]$ and its 2π-periodic extension $\tilde{f}(x)$ by

$$\tilde{f}(x) = f(x - 2k\pi) \quad \text{for } x \in (2k\pi, 2(k+1)\pi], \ \forall k \in \mathbb{Z}.$$

Sketch $\tilde{f}$, find its Fourier series, and discuss the pointwise convergence. Finally, if the Fourier series converges at the point $x = 0$, what value does it limit to?

(4) Consider the same function $f(x) = x^2$ but only on $[0, \pi]$. Extending it to an odd function $f_1(x)$ on $[-\pi, \pi]$, then further extend f_1 to a 2π-periodic function $\tilde{f}_1$ as usual. Sketch $\tilde{f}_1$, find its Fourier series, and discuss the pointwise convergence. Finally, if the Fourier series converges at the point $x = 0$, what value does it limit to? Is it the same value as in the problem (3)?

(End)