

THE CHINESE UNIVERSITY OF HONG KONG
DEPARTMENT OF MATHEMATICS

MATH1010G University Mathematics 2014-2015
Test 2, 17 Mar, 2015

- Time allowed: 45 minutes
- Answer all questions.
- Show your work clearly and concisely in your answer book.
- Write down your name and student ID number on the front page of your answer book.
- You are allowed to use a calculator in this test.

1. By using L'Hôpital Rule, find

(a) $\lim_{x \rightarrow 0} \frac{e^x - e^{-x} - 2 \sin x}{x^3}$

(b) $\lim_{x \rightarrow 0} (e^x + x)^{1/x}$

(20 points)

2. By using implicit differentiation, find $\frac{d}{dx} \tan^{-1} x$.

(15 points)

3. Write down the Taylor polynomial $P_3(x)$ of degree 3 generated by $f(x) = \ln(1-x)$ at 0, and hence approximate $\ln 0.99$.

(15 points)

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4. (a) Let $0 < k < 1$. Show that

$$kt + (1 - k) \geq t^k \text{ for all } t > 0.$$

- (b) Hence, deduce that

$$kr + (1 - k)s \geq r^k s^{1-k} \text{ for all } r, s > 0.$$

(20 points)

5. Let $f(x) = -xe^{-x^2}$, where x is a real number.

- (a) Find $f'(x)$ and $f''(x)$

- (b) Find the range of x such that

(i) $f'(x) > 0$

(ii) $f'(x) < 0$

(iii) $f''(x) > 0$

(iv) $f''(x) < 0$

- (c) Find the local extrema and saddle points, if any.

- (d) Find the points of inflection, if any.

- (e) Find the asymptotes of the graph of $f(x)$, if any.

- (f) Sketch the graph of $f(x)$.

(30 points)