MATH1010 University Mathematics
Derivatives

1. Use the definition of derivative to find $\dfrac{dy}{dx}$ of the following functions.

   (a) $y = \dfrac{2}{\sqrt{x}}$
   (b) $y = (\ln x)^2$
   (c) $y = \cos^3 x$
   (d) $y = \tan 2x$
   (e) $y = x^2 e^{3x}$
   (f) $y = \dfrac{\sin x}{x}$

2. Find $\dfrac{dy}{dx}$ of the following functions.

   (a) $y = e^{4x} \sin x$
   (b) $y = \sqrt{x} \cos 3x$
   (c) $y = x^2 \tan x$
   (d) $y = \ln 2x \tan^2 x$
   (e) $y = \dfrac{1 - \cos x}{1 + \cos x}$
   (f) $y = \dfrac{e^{\sqrt{x}}}{x}$
   (g) $y = \ln(1 + \sin 2x)$
   (h) $y = \ln \sec x$
   (i) $y = \sec^2(1 + x^3)$
   (j) $y = \dfrac{x}{\sqrt{1 + x^2}}$
   (k) $y = 3^{x+2}$
   (l) $y = \log_2(1 + x^2)$
   (m) $y = \sin(\ln x)$
   (n) $y = e^{e^x}$
   (o) $y = \ln(x + \sqrt{1 + x^2})$
   (p) $y = \dfrac{\sin 5x}{\sqrt{1 + 2x}}$
   (q) $y = \sqrt{\sin \sqrt{x}}$
   (r) $y = \dfrac{1}{\sqrt{1 - x^2}}$
   (s) $y = \dfrac{1}{(\ln x)^3}$
   (t) $y = \tan\left(\dfrac{1 - x}{1 + x}\right)$
   (u) $y = \ln\left(\dfrac{e^x - 1}{e^x + 1}\right)$
   (v) $y = \sqrt{\dfrac{1 + \sqrt{x}}{1 - \sqrt{x}}}$

3. Find $\dfrac{dy}{dx}$ of the following implicit functions.

   (a) $x^2 - y^2 = 4$
   (b) $4x^2 y + 5xy = 3$
   (c) $\sqrt{x} + \sqrt{y} = 3$
   (d) $(x^2 + y^3)^2 = 5x^3 y^2$
   (e) $y \sin x - x \cos y = 0$
   (f) $\cos(x^2 - y^2) = xy$

4. Find $\dfrac{dy}{dx}$ of the following functions.

   (a) $y = 2^{\cos x}$

   (b) $y = x^{2x}$

   (c) $y = \dfrac{(1-x)^{\frac{3}{2}}\sqrt{1+x}}{(1+x^2)^2}$

   (d) $y = \sqrt{\dfrac{(2x+1)(3x-1)}{x^2+1}}$

   (e) $y = x^{\frac{1}{x^2}}$

   (f) $y = (\ln x)^x$

5. Find $\dfrac{dy}{dx}$ of the following functions.

   (a) $y = (1+x^2)\tan^{-1} x$

   (b) $y = \tan^{-1}\left(\dfrac{x}{\sqrt{1-x^2}}\right)$

   (c) $y = (\sin^{-1} x)^2$

   (d) $y = \cos^{-1}(2\cos x)$

6. Find $\dfrac{d^2y}{dx^2}$ of the following.

   (a) $y = x^3 e^{2x}$

   (b) $y = \ln(\sec x + \tan x)$

   (c) $y = \ln(x - \sqrt{1+x^2})$

   (d) $y = \sin^{-1}\sqrt{1-x^2}$, $0 < x < 1$

   (e) $x = y^2 + y + 1$

   (f) $x^2 + y^2 = 1$

   (g) $\sqrt{x} + \sqrt{y} = 1$

   (h) $xy - y^2 = 3$

Answers:

1. (a) $-\frac{1}{x^{\frac{3}{2}}}$

   (b) $\frac{2\ln x}{x}$

   (c) $-3\cos^2 x \sin x$

   (d) $2\sec^2 2x$

   (e) $(3x^2 - 2x)e^{3x}$

   (f) $\frac{x\cos x - \sin x}{x^2}$

2. (a) $e^{4x}(4\sin x + \cos x)$

   (b) $\frac{\cos 3x - 6x\sin 3x}{2\sqrt{x}}$

   (c) $x^2\sec^2 x + 2x\tan x$

   (d) $2\ln 2x \sec^2 x \tan x + \dfrac{\tan^2 x}{x}$

   (e) $\frac{2\sin x}{(1+\cos x)^2}$

   (f) $\frac{(\sqrt{x}-2)e^{\sqrt{x}}}{2x^2}$

   (g) $\frac{2\cos 2x}{1+\sin 2x}$

   (h) $\tan x$

   (i) $6x^2\sec^2(1+x^3)\tan(1+x^3)$

   (j) $3^{x+2}\ln 3$

   (k) $\frac{1}{(1+x^2)^{\frac{3}{2}}}$

   (l) $\frac{2x}{(1+x^2)\ln 2}$

(m) $\frac{\cos(\ln x)}{x}$

(n) e^{e^x+x}

(o) $\frac{1}{\sqrt{1+x^2}}$

(p) $\frac{5\cos 5x-(1+2x)\sin 5x}{(1+2x)^{\frac{3}{2}}}$

(q) $\frac{\cos\sqrt{x}}{4\sqrt{x\sin\sqrt{x}}}$

3. (a) $\frac{x}{y}$

(b) $-\frac{8xy+5y}{4x^2+5x}$

(c) $-\frac{\sqrt{y}}{\sqrt{x}}$

4. (a) $-2^{\cos x}(\ln 2)\sin x$

(b) $2x^{2x}(1+\ln x)$

(c) $\frac{(2x^3-x^2-6x-1)\sqrt{1-x}}{(1+x^2)^3\sqrt{1+x}}$

5. (a) $1+2x\tan^{-1}x$

(b) $\frac{1}{\sqrt{1-x^2}}$

6. (a) $(4x^3+12x^2+6x)e^{2x}$

(b) $\sec x\tan x$

(c) $\frac{x}{(1+x^2)^{\frac{3}{2}}}$

(d) $-\frac{x}{(1-x^2)^{\frac{3}{2}}}$

(r) $\frac{x}{(1-x^2)^{\frac{3}{2}}}$

(s) $-\frac{3}{x(\ln x)^4}$

(t) $-\frac{1}{(1+x)^2}\sec^2(\frac{1-x}{1+x})$

(u) $\frac{2e^x}{e^{2x}-1}$

(v) $\frac{1}{2x^{\frac{1}{2}}(1+x)^{\frac{1}{2}}(1-x)^{\frac{3}{2}}}$

(d) $\frac{4x^3+4xy^3-15x^3y^2}{10x^3y-6x^2y^2-6y^4}$

(e) $\frac{\cos y-y\cos x}{x\sin y+\sin x}$

(f) $\frac{x+2x\sin(x^2-y^2)}{2y\sin(x^2-y^2)-x}$

(d) $\frac{-x^2+14x+1}{2(2x+1)^{\frac{1}{2}}(3x-1)^{\frac{1}{2}}(x^2+1)^{\frac{3}{2}}}$

(e) $-x^{\frac{1}{x}-3}(2\ln x-1)$

(f) $(\ln x)^{x-1}+(\ln x)^x\ln(\ln x)$

(c) $\frac{2\sin^{-1}x}{\sqrt{1-x^2}}$

(d) $\frac{2\sin x}{\sqrt{1-4\cos^2 x}}$

(e) $-\frac{2}{(2y+1)^3}$

(f) $-\frac{1}{y^3}$

(g) $\frac{1}{x^{\frac{3}{2}}}$

(h) $\frac{2y^2-2xy}{(2y-x)^3}$