THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MMAT5520 Differential Equation & Linear Algebra

Assignment 4
Due date: 1 Dec (Thursday)

Exercise 5.2

1. Diagonalize the following matrices.

   (b) $\begin{pmatrix} 3 & -2 \\ 4 & -1 \end{pmatrix}$ (d) $\begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 6 & 11 & 6 \end{pmatrix}$ (e) $\begin{pmatrix} 3 & -2 & 0 \\ 0 & 1 & 0 \\ -4 & 4 & 1 \end{pmatrix}$

2. Show that that following matrices are not diagonalizable.

   (a) $\begin{pmatrix} 3 & 1 \\ -1 & 1 \end{pmatrix}$ (b) $\begin{pmatrix} -1 & 1 & 0 \\ -4 & 3 & 0 \\ 1 & 0 & 2 \end{pmatrix}$

7. Let $\mathbf{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ be a 2×2 matrix. Show that if $(a-d)^2 + 4bc \neq 0$, then $\mathbf{A}$ is diagonalizable.

10. Prove that if $\mathbf{A}$ is a non-singular matrix, then for any matrix $\mathbf{B}$, we have $\mathbf{AB}$ is similar to $\mathbf{BA}$.

Exercise 5.3

1. Compute $\mathbf{A}^5$ where $\mathbf{A}$ is the given matrix.

   (a) $\begin{pmatrix} 5 & -6 \\ 3 & -4 \end{pmatrix}$ (d) $\begin{pmatrix} 1 & -5 \\ 1 & -1 \end{pmatrix}$ (e) $\begin{pmatrix} 1 & 2 & -1 \\ 2 & 4 & -2 \\ 3 & 6 & -3 \end{pmatrix}$

Exercise 5.4

1. Find the minimal polynomial of $\mathbf{A}$ where $\mathbf{A}$ is the matrix given below. Then express $\mathbf{A}^4$ and $\mathbf{A}^{-1}$ as a polynomial in $\mathbf{A}$ of smallest degree.

   (a) $\begin{pmatrix} 5 & -4 \\ 3 & -2 \end{pmatrix}$

   (b) $\begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}$

   (d) $\begin{pmatrix} -1 & 1 & 0 \\ -4 & 3 & 0 \\ 1 & 0 & 2 \end{pmatrix}$

   (e) $\begin{pmatrix} 3 & 1 & 1 \\ 2 & 4 & 2 \\ -1 & -1 & 1 \end{pmatrix}$