THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MATH1010D&E (2016/17 Term 1)
University Mathematics
Tutorial 12

This is the last tutorial. We will provide exercises that include most of the topics in this course. We will mainly focus on computational problems, you may take it as a revision. Good luck to your exam.

Problems that may be demonstrated in class :

Q1. For each of the following sequence, compute the limit if it exists.

(a) $\lim_{n\to\infty} \frac{3^n + (-2)^{n+1}}{3^{n-2} - 2^{2n-1}}$ (b) $\lim_{n\to\infty} \frac{\ln^2(n+1)}{(n-1)^2}$ (c) $\lim_{n\to\infty} \frac{n^2 + n\sin n}{n^2}$

Q2. Compute the following limits if exist.

(a) $\lim_{x\to 0} x^2 \cos\frac{1}{x}$ (b) $\lim_{x\to\infty} \frac{2e^{3x} + 2e^x + 1}{3e^{3x} + 3}$ (c) $\lim_{x\to\infty} \frac{x^5 + 2x + 3}{x^4 + 3}$ (d) $\lim_{x\to 3} \frac{x^2 - 9}{x - 3}$

Q3. Let

$$f(x) = \begin{cases} x^2 - 2 & \text{if } x < 1 \\ Ax - 4 & \text{if } x \geq 1 \end{cases}$$

Find the value of A if f is continuous.

Q4. Determine whether the following functions are differentiable.

(a) $f(x) = |x + 2|$ (b) $f(x) = \begin{cases} x^2 + 1 & \text{if } x < 1 \\ 3 & \text{if } x = 1 \\ 2\sqrt{2x - 1} & \text{if } x > 1 \end{cases}$

Q5. Compute $f'(x)$.

(a) $f(x) = x^2 e^x$ (b) $f(x) = \frac{x^2 + 1}{x^3 + 2}$ (c) $f(x) = \int_0^x (3t^2 + 3)dt$

(d) $f(x) = \int_{-x^3}^{e^{2x}} t dt$

Q6. Prove that the equation $x^5 + 7x - 2 = 0$ has exactly one real root.

Q7. Find the Taylor series of the following functions at $x = 0$.

(a) $f(x) = x^4 e^{-x}$ (b) $f(x) = \frac{2x}{(1 + x)^2}$

Q8. (a) Let $f(x) = \sin x$, using Taylor theorem to show that

$$\frac{599}{6000} - \frac{0.1^4}{4!} \leq f(0.1) \leq \frac{599}{6000} + \frac{0.1^4}{4!}$$

(b) If we use the Taylor polynomial of f of degree n to approximate f, find one n such that the absolute error is less than 10^{-7}.

Q9. Compute the following integral.

(a) (1.1.14) $\int \frac{dx}{1+e^x}$ (b) (1.2.20) $\int e^{2x}\cos 3x dx$ (c) (1.3.18) $\int \sin^2 x \cos^4 x dx$

(d) (1.5.3) $\int \sqrt{\frac{1+x}{1-x}} dx$ (e) (1.6.10) $\int \frac{2x^3-4x^2-x-3}{x^2-2x-3} dx$ (f) (1.7.3) $\int \frac{dx}{\sin x \cos^4 x}$

(g) (1.8.36) $\int_0^1 \frac{dx}{1+\sqrt{x}}$

Q10. (1.3.4) Prove the following reduction formula:

$$I_n = \int \frac{1}{\sin^n x} dx;\ I_n = -\frac{\cos x}{(n-1)\sin^{n-1} x} + \frac{n-2}{n-1} I_{n-2},\ n \geq 2$$