THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics
MATH4250 Game Theory, 2018-2019 Term 2
Mid-term Examination
Time allowed: 90 mins
Answer all questions. Full marks: 50

1. (8 marks) In the Wythoff's game, there are 2 piles of chips. In each turn, a player may either remove any positive number of chips from one of the piles, or remove the same positive number of chips from both piles. The player removing the last chip wins.

   (a) If $(a, 15)$ is a P-position, find a.

   (b) If $(b, b+60)$ is a P-position, find b.

   (c) Find all winning moves from the positions $(15, 23)$ and $(100, 160)$.

2. (8 marks) Consider the following three games.

   Game 1: At least three game (In each turn, a player removes at least 3 chips. The terminal positions are $0, 1, 2$.)

   Game 2: Subtraction game with subtraction set $S = \{1, 2, 3, 4, 5, 6\}$

   Game 3: 1-pile nim

   Let g_1, g_2, g_3 be the Sprague-Grundy functions of the three games respectively. Let G be the sum of the three games and g be the Sprague-Grundy function of G.

   (a) Find $g_1(11), g_1(12), g_1(13)$.

   (b) Find $g(16, 13, 7)$.

   (c) Find all winning moves of G from the position $(16, 13, 7)$.

3. (6 marks) Aaron puts a chip in either his left hand or right hand. Ben guesses where the chip is. If Ben guesses the left hand, he receives \$2 from Aaron if he is correct and pays \$4 to Aaron if he is wrong. If Ben guesses the right hand, he receives \$1 from Aaron if he is correct and pays \$3 to Aaron if he is wrong.

   (a) Write down the payoff matrix of Aaron. (Use order of strategies: Left, Right.)

   (b) Find the maximin strategy for Aaron, the minimax strategy for Ben and the value of the game.

4. (8 marks) Let

$$A = \begin{pmatrix} 2 & 1 & 0 & 2 & -3 \\ 1 & -1 & 2 & 4 & 3 \\ 3 & 4 & 1 & -1 & -2 \end{pmatrix}$$

(a) Write down the reduced matrix obtained by deleting all dominated rows and columns of A.

(b) Use the reduced matrix to solve the game matrix A.

5. (8 marks) Use simplex method to solve the game matrix

$$\begin{pmatrix} 3 & 1 & -2 \\ 2 & 3 & -1 \\ -1 & -2 & 2 \end{pmatrix}$$

6. (12 marks) For positive integer k, define

$$A_k = \begin{pmatrix} 4k-3 & -(4k-2) \\ -(4k-1) & 4k \end{pmatrix}.$$

(a) Solve A_k, that is, find the maximin strategy, minimax strategy and value of A_k in terms of k.

(b) Let $r_1, r_2, \cdots, r_n > 0$ be positive real numbers. Using the principle of indifference, or otherwise, find, in terms of $r_1, r_2, \cdots, r_n$, the value of

$$D = \begin{pmatrix} \frac{1}{r_1} & 0 & 0 & \cdots & 0 \\ 0 & \frac{1}{r_2} & 0 & \cdots & 0 \\ 0 & 0 & \frac{1}{r_3} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \frac{1}{r_n} \end{pmatrix}.$$

(c) Find, with proof, the value of the matrix

$$A = \begin{pmatrix} A_1 & 0 & 0 & \cdots & 0 \\ 0 & A_2 & 0 & \cdots & 0 \\ 0 & 0 & A_3 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & A_{25} \end{pmatrix}.$$

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