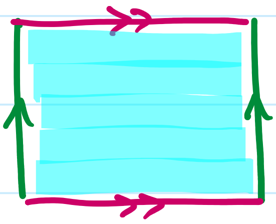
Lect13-20180228

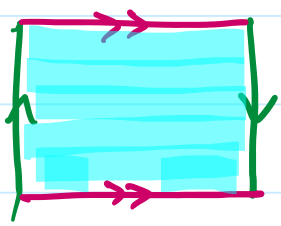
Wednesday, 28 February 2018 11:30 AM

Quotient Spaces on $[0,1]\times[0,1]$

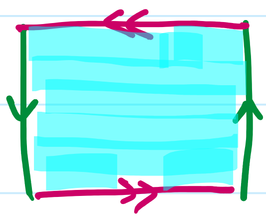
Torus



Klein Bottle

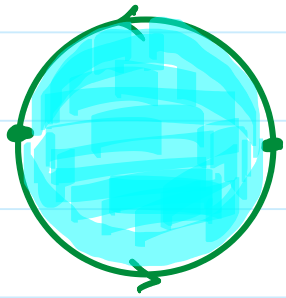


Projective Plane

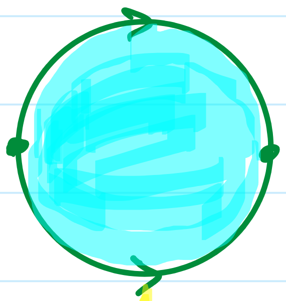


$\parallel$

...... $=$



Examples. Two boundary identification of $\mathbb{D}^2$



and

$\mathbb{D}^2/\sim$ where

$z_1 \sim z_2$ if

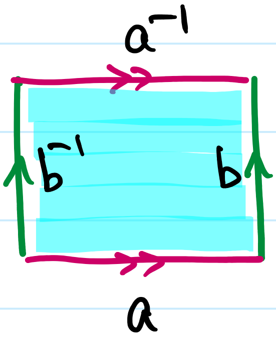
$|z_1| = |z_2| = 1$

What are they ?

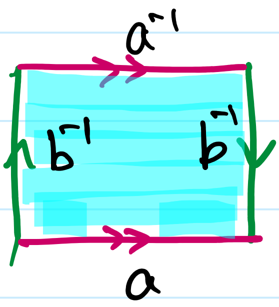
餃子 $=$ S^2 $=$ 义燒包

Notation

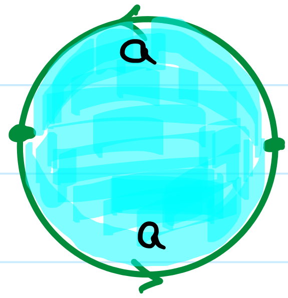
Torus



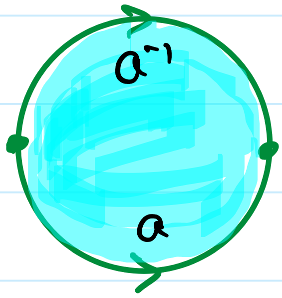
Klein Bottle



Projective Plane



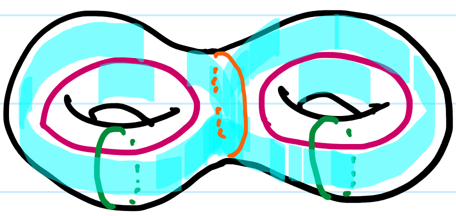
Sphere



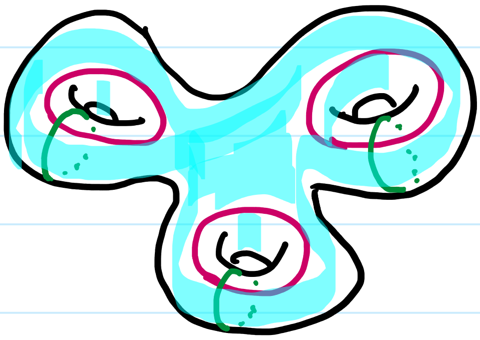
A sequence of symbols, each exists twice, on the boundary S^1 of $\mathbb{D}^2$.

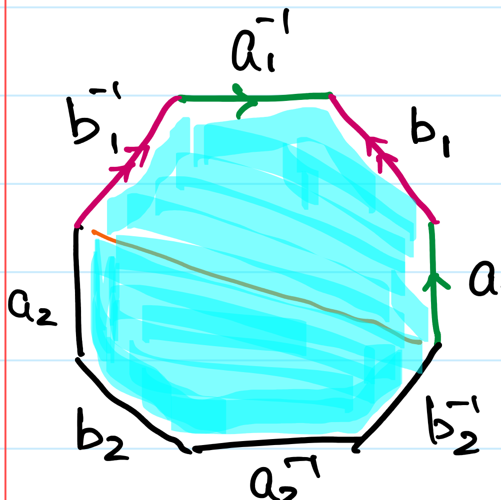
Examples. Surfaces of genus $= n$

$n=2$



$n=3$





on 12-gon,

$$a_1b_1a_1^{-1}b_1^{-1}a_2b_2a_2^{-1}b_2^{-1}a_3b_3a_3^{-1}b_3^{-1}$$

Classification of Surfaces

A compact surface without boundary is homeomorphic to one case below.

(1) Sphere S^2

(2) $(4n\text{-gon})/\sim$ where $\sim$ is determined by

$$a_1 b_1 a_1^{-1} b_1^{-1} \cdots \cdots a_n b_n a_n^{-1} b_n^{-1}$$

orientable cases

(3) $(2n\text{-gon})/\sim$ where $\sim$ is determined by

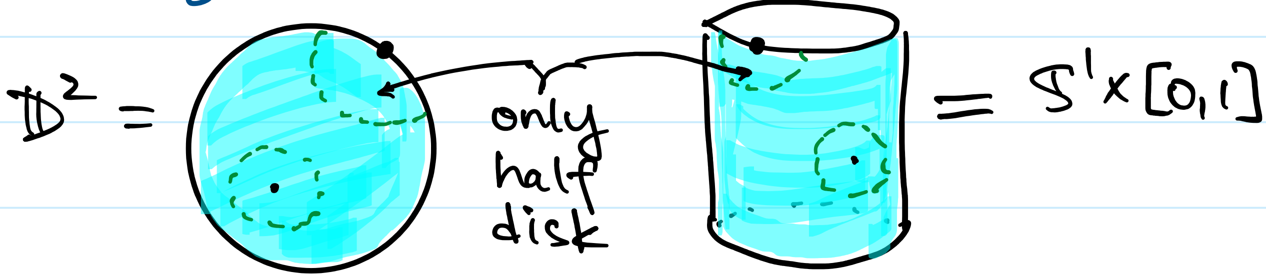
$$a_1^2 a_2^2 \cdots a_n^2$$

non-orientable cases

What does it mean by,

- compact
- without boundary
- orientable ?

Boundary exists



On surfaces without boundary (S^2 or Torus), every point has basic neighborhoods that look like full disks.

Non-compact

$\mathbb{R}^2$ or $S^1 \times (0,1)$ or $\mathbb{R}$ or (a,b)

In them, some infinite sequences may not have cluster point. We will discuss in the sense of Heine-Borel later.

Non-orientable

Möbius strip, Klein Bottle

They have only "one side", cannot be painted by two colors. Or, they cannot have a consistent "normal".

Surfaces

你明的 intuitively.

Definition. Given $(X, \mathcal{T}_X)$, $(Y, \mathcal{T}_Y)$. The **disjoint union**, $X \sqcup Y = X \times \{0\} \cup Y \times \{1\}$ is given the topology generated by

$$\{U \times \{0\} : U \in \mathcal{T}_X\} \cup \{V \times \{1\} : V \in \mathcal{T}_Y\}$$

Intuitively, this is putting X, Y together side b side.

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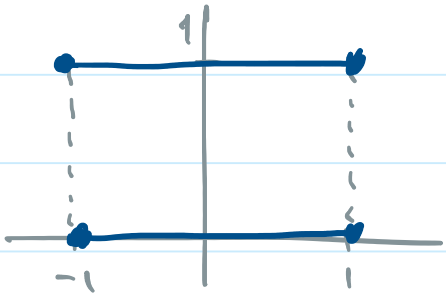
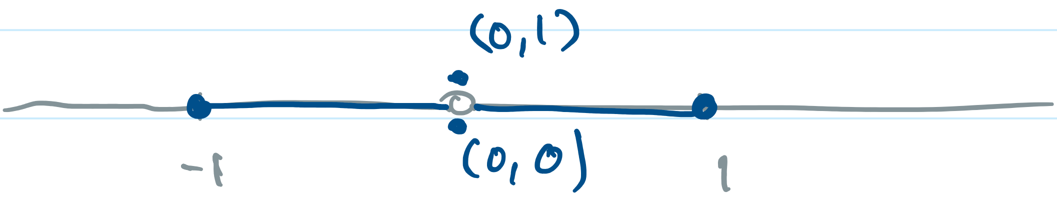
Example. $X = Y = [-1,1] \subset (\mathbb{R}, \text{std})$

$X \sqcup Y =$



Very Important Example On $[-1,1] \sqcup [-1,1]$, identify $(x,0)$ with $(x,1)$ $\forall\, x \neq 0$

The illustrative picture is



Exercise. Every pair of nbhds at $(0,0)$ and $(0,1)$ will have common intersection of the form $(-\varepsilon, 0) \cup (0, \varepsilon)$, $\varepsilon > 0$.

Consequence. The resulting space is non-Hausdorff.