MATH1030 Further examples on matrix inverse and row operations

1. Consider each of the square matrices below. Apply row-operations to determine whether it is non-singular or singular. Where it is non-singular, also find a matrix inverse for the matrix concerned:

   (a) $\begin{bmatrix} 3 & 5 \\ 2 & 3 \end{bmatrix}$

   (b) $\begin{bmatrix} 5 & 3 \\ 4 & 2 \end{bmatrix}$

   (c) $\begin{bmatrix} 2 & -3 \\ 1 & 3 \end{bmatrix}$

   (d) $\begin{bmatrix} -2 & 6 \\ 3 & -9 \end{bmatrix}$

   (e) $\begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{bmatrix}$.

   (f) $\begin{bmatrix} 1 & -2 & 2 \\ 2 & -3 & 6 \\ 1 & 1 & 7 \end{bmatrix}$.

   (g) $\begin{bmatrix} 1 & 2 & -4 \\ -1 & -1 & 5 \\ 2 & 7 & -3 \end{bmatrix}$.

   (h) $\begin{bmatrix} 1 & 3 & -4 \\ 1 & 5 & -1 \\ 3 & 13 & 6 \end{bmatrix}$.

   (i) $\begin{bmatrix} 1 & 1 & 1 & 5 \\ 1 & 3 & 2 & 9 \\ 1 & 1 & -1 & 1 \\ 1 & 2 & 1 & 6 \end{bmatrix}$.

   (j) $\begin{bmatrix} 1 & 1 & 2 & 1 \\ 0 & -2 & 0 & 0 \\ 1 & 2 & 1 & -2 \\ 0 & 3 & 2 & 1 \end{bmatrix}$.

2. Let $\alpha, \beta, \gamma, \delta$ be real numbers.

   (a) Let A be the (3×3)-square matrix given by $A = \begin{bmatrix} 1 & \alpha & \alpha^2 \\ 1 & \beta & \beta^2 \\ 1 & \gamma & \gamma^2 \end{bmatrix}$.

   Show that A is non-singular and invertible if and only if α, β, γ are pairwise distinct.

   (b) Let B be the (4×4)-square matrix given by $A = \begin{bmatrix} 1 & \alpha & \alpha^2 & \alpha^3 \\ 1 & \beta & \beta^2 & \beta^3 \\ 1 & \gamma & \gamma^2 & \gamma^3 \\ 1 & \delta & \delta^2 & \delta^3 \end{bmatrix}$.

   Show that B is non-singular and invertible if and only if $\alpha, \beta, \gamma, \delta$ are pairwise distinct.

Answer.

1. (a) Non-singular. Matrix inverse: $\begin{bmatrix} -3 & 5 \\ 2 & -3 \end{bmatrix}$.

   (b) Non-singular. Matrix inverse: $\begin{bmatrix} -1 & 3/2 \\ 2 & 5/2 \end{bmatrix}$.

   (c) Non-singular. Matrix inverse: $\begin{bmatrix} 1/3 & 1/3 \\ -1/9 & 2/9 \end{bmatrix}$.

   (d) Singular. No matrix inverse.

   (e) Non-singular. Matrix inverse: $\begin{bmatrix} -11 & 2 & 2 \\ -4 & 0 & 1 \\ 6 & -1 & -1 \end{bmatrix}$.

   (f) Non-singular. Matrix inverse: $\begin{bmatrix} 27 & -16 & 6 \\ 8 & -5 & 2 \\ -5 & 3 & -1 \end{bmatrix}$.

   (g) Non-singular. Matrix inverse: $\begin{bmatrix} -16 & -11 & 3 \\ 7/2 & 5/2 & -1/2 \\ -5/2 & -3/2 & 1/2 \end{bmatrix}$.

   (h) Singular. No matrix inverse.

   (i) Singular. No matrix inverse.

   (j) Non-singular. Matrix inverse. $\begin{bmatrix} 1 & -1 & 0 & -1 \\ 0 & -1/2 & 0 & 0 \\ -1/5 & 1 & 1/5 & 3/5 \\ 2/5 & -1/2 & -2/5 & -1/5 \end{bmatrix}$.

2. ——