

## Examples

Determine all solutions (if any) for the systems of linear equations below:

$$\textcircled{a} (\$) \begin{cases} x_1 - 2x_2 + 3x_3 - 4x_4 = 5 \\ 2x_1 - 3x_2 + 4x_3 - 5x_4 = 6 \\ 3x_1 - 4x_2 + 5x_3 - 6x_4 = 10 \end{cases}$$

$$\textcircled{b} (\text{T}) \begin{cases} 2x_1 - 5x_2 + 3x_3 - 4x_4 + 2x_5 = 4 \\ 3x_1 - 7x_2 + 2x_3 - 5x_4 + 4x_5 = 9 \\ 5x_1 - 10x_2 - 5x_3 - 4x_4 + 7x_5 = 22 \end{cases}$$

$$\textcircled{c} (\text{U}) \begin{cases} x_1 + 2x_2 = 3 \\ x_1 + 2x_3 = 3 \\ x_2 + 2x_4 = 3 \\ x_3 + 2x_4 = 3 \end{cases}$$

Answer.

(a) From (S), you will reach 
$$\begin{cases} x_1 - 2x_2 + 3x_3 - 4x_4 = 5 \\ x_2 - 2x_3 + 3x_4 = -4 \\ 0 = 3 \end{cases}$$

There is no solution for (S).

(b) From (T), you will reach 
$$\begin{cases} x_1 - 11x_3 + 15x_5 = 26 \\ x_2 - 5x_3 + 8x_5 = 12 \\ x_4 - 3x_5 = -3 \end{cases}$$

The solutions of (T) is described by

$$(x_1, x_2, x_3, x_4, x_5) = (26 + 11s - 15t, 12 + 5s - 8t, s, -3 + 3t, t)$$

where  $s, t$  are arbitrary numbers.

(c) From (U), you will reach 
$$\begin{cases} x_1 - 4x_4 = -3 \\ x_2 + 2x_4 = 3 \\ x_3 + 2x_4 = 3 \\ 0 = 0 \end{cases}$$

The solutions of (U) is described by

$$(x_1, x_2, x_3, x_4) = (-3 + 4t, 3 - 2t, 3 - 2t, t)$$

where  $t$  is an arbitrary number.