# Suggested Solution to Quiz 1

Feb 6, 2018

1. (5 points) For each of the following equations, state the order, type and whether it is nonlinear, linear inhomogeneous, or linear homogeneous:

   (a) $\partial_t u - 2\partial_x^2 u + \sin x = 0$

   (b) $\partial_t u - 5\partial_x u + u^2 = 0$

   (c) $\partial_{xy}^2 u = x$

   (d) $2\partial_x^2 u + \sin x \partial_{xy}^2 u + \partial_y^2 u = 0$

   **Solution:**

   (a) 2nd order(0.25points); parabolic equation(0.5points); linear inhomogeneous(0.5points).

   (b) 1st order(0.25points); hyperbolic/transport(0.5points); nonlinear(0.5points)

   (c) 2nd order(0.25points); hyperbolic equation(0.5points); linear inhomogeneous(0.5points).

   (d) 2nd order(0.25points); elliptic equation(0.5points); linear homogeneous(0.5points).

2. (5 points) Solve the linear equation

$$\partial_x u + x\partial_y u = 0$$

   with the following conditions:

   (a) $u(0, y) = y^2$.

   (b) $u(x, 0) = x^2$

   **Solution:** The characteristic curves satisfy the ODE:

$$\frac{dx}{1} = \frac{dy}{x}$$

   Hence the characteristic curves are

$$y = \frac{1}{2}x^2 + C \quad \text{(1point)}$$

   Therefore, the general solution is

$$u(x, y) = f(y - \frac{1}{2}x^2) \quad \text{(1point)}$$

   where $f$ is an arbitrary function.

   (a) By $u(0, y) = y^2$, we have $u(0, y) = f(y) = y^2$.Therefore $u(x, y) = (y - \frac{1}{2}x^2)^2$ on $\mathbb{R}^2$.(1 point)

   (b) By $u(x, 0) = x^2$, we have $u(x, 0) = f(-\frac{1}{2}x^2) = x^2$ which implies $f(z) = -2z$. Therefore $u(x, y) = -2(y - \frac{1}{2}x^2) = x^2 - 2y$ (1 point) on the domain $\{(x, y) : y - \frac{1}{2}x^2 \leq 0\}$(1 point).On the domain $\{(x, y) : y - \frac{1}{2}x^2 > 0\}$, the solution of $u(x, y)$ can not be determined uniquely.