Date

MATH2230 Complex Variables and Application
HW 6 & 7

Due on Oct. 31

Sect. 46 No. 1, 2, 3, 4
Sect. 47 No. 1, 4
Sect. 49 No. 2
Sect. 53 No. 1

Sect. 46 $\Big\{$ For the functions f and contours C in Exercises 1 through 8, use paramet representations for C, or legs of C, to evaluate $\int_C f(z)\,dz$

1. $f(z) = \frac{z+2}{z}$ and C is
   (a) the semicircle $z = 2e^{i\theta}$ $(0 \le \theta \le \pi)$;
   (b) the semicircle $z = 2e^{i\theta}$ $(\pi \le \theta \le 2\pi)$;
   (c) the circle $z = 2e^{i\theta}$ $(0 \le \theta \le 2\pi)$.

   Ans. (a) $-4+2\pi i$; (b) $4+2\pi i$; (c) $4\pi i$.

2. $f(z) = z-1$ and C is the arc from $z=0$ to $z=2$ consisting of
   (a) the semicircle $z = 1+e^{i\theta}$ $(\pi \le \theta \le 2\pi)$;
   (b) the segment $z = x$ $(0 \le x \le 2)$ of the real axis.

   Ans. (a) 0; (b) 0.

3. $f(z) = \pi \exp(\pi \bar{z})$ and C is the boundary of the square with vertices at the points $0, 1, 1+i$, and i, the orientation of C being in the counterclockwise direction.
   Ans. $4(e^{\pi}-1)$.

4. $f(z)$ is defined by means of the equations

$$f(z) = \begin{cases} 1 & \text{when } y<0, \\ 4y & \text{when } y>0, \end{cases}$$

   and C is the arc from $z=-1-i$ to $z=1+i$ along the curve $y=x^3$.
   Ans. $2+3i$.

Sect. 47.

1. Without evaluating the integral, show that

   (a) $\left| \int_C \frac{z+4}{z^3-1}\,dz \right| \le \frac{6\pi}{7}$; (b) $\left| \int_C \frac{dz}{z^2-1} \right| \le \frac{\pi}{3}$

   when C is the arc that use in Example 1, Sec. 47.

4. Let C_R denote the upper half of the circle $|z| = R$ $(R>2)$, taken in the counterclockwis direction. Show that

$$\left| \int_{C_R} \frac{2z^2-1}{z^4+5z^2+4}\, dz \right| \le \frac{\pi R(2R^2+1)}{(R^2-1)(R^2-4)}.$$

Then, by dividing the numerator and denominator on the right here by R^4, show that the value of the integral tends to zero as R tends to infinity. (Compare with Example 2 in Sec. 47)

Sect. 49

2. By finding an antiderivative, evaluate each of these integrals, where the path is any contour between the indicated limits of integration:

(a) $\int_0^{1+i} z^2\, dz$; (b) $\int_0^{\pi+2i} \cos\left(\frac{z}{2}\right) dz$; (c) $\int_1^3 (z-2)^3\, dz$.

Ans. (a) $\frac{2}{3}(-1+i)$; (b) $e+\frac{1}{e}$; (c) 0.

Sect. 53.

1. Apply the Cauchy-Goursat theorem to show that

$$\int_C f(z)\, dz = 0$$

when the contour C is the unit circle $|z|=1$, in either direction, and when

(a) $f(z) = \frac{z^2}{z+3}$; (b) $f(z) = ze^{-z}$; (c) $f(z) = \frac{1}{z^2+2z+2}$;

(d) $f(z) = \text{sech}\, z$; (e) $f(z) = \tan z$; (f) $f(z) = \text{Log}\,(z+2)$.