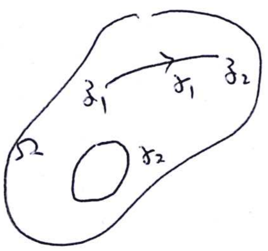
Lecture 11.

In this lecture, we introduced anti-derivative.

and show the following equivalence



(a) f has anti-derivative function in Ω.

i.e. $\exists\ F$ analytic s.t $F'(z) = f(z)$

(b) $\forall\ \gamma_1$ curve.

$$\int_{\gamma_1} f(z)\,dz = F(z_2) - F(z_1)$$

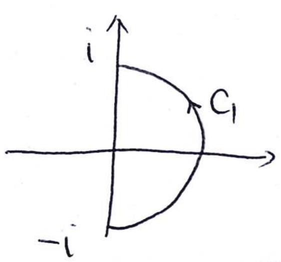
(c). $\forall\ \gamma_2$ closed curve

$$\int_{\gamma_2} f(z)\,dz = 0.$$

while you calculate integral by anti-derivative, be sure

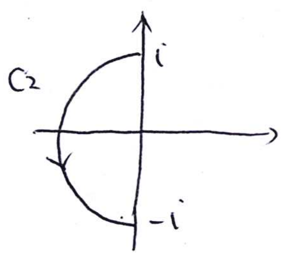
anti-function F is analytic at all pts on curve

ex:



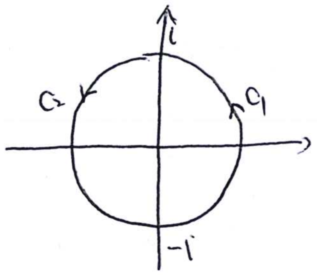
$$\int_{C_1} \frac{1}{z}\,dz = \log(i) - \log(-i)$$

$$-\pi < \arg z < \pi.$$



$$\int_{C_2} \frac{1}{z}\,dz = \log(-i) - \log i$$

$$0 < \arg z < 2\pi.$$



$$\int_{C_1+C_2} \frac{1}{z}\,dz = 2\pi i$$

$\therefore$ $\log z$ is not analytic on whole curve.

this is the fact since $\log z = \log r + i\theta$ and θ

is even not continuous on the branch cut.