lecture 6.

ex: f and $\bar{f}$ are analytic in D $\Rightarrow$ $f \equiv$ Const.

ex. if $|f(z)|^2 =$ const $\Rightarrow$ f is a Const.

prop: if f, g, h are analytic. then $\frac{fg}{h}$ is analytic except at where $h = 0$.

ex. $f(z) = \dfrac{z^2+3}{(z+1)(z^2+5)}$ is analytic except at -1, $\pm\sqrt{5}i$.

ex. $f(z) = \sin x \cosh y + i \cos x \sinh y$

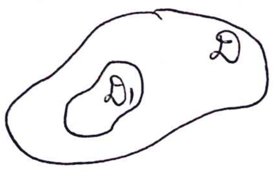
Check analyticity of f.

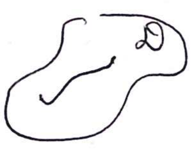
if f is analytic $\Rightarrow$ $f = u + iv$ satisfies

$$\partial_{xx} u + \partial_{yy} u = \partial_{xx} v + \partial_{yy} v = 0.$$

Thm: D is connected open set. f is analytic in D.

then the following holds. if $f \equiv 0$ in D_1, or $f \equiv 0$ on a continuous curve in D then $f \equiv 0$ in D





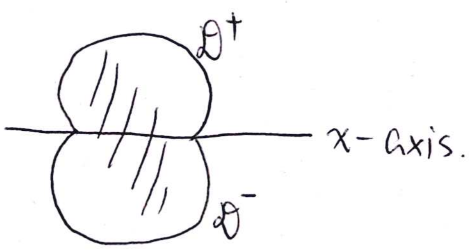
Def: $\mathcal{D}$ is a connected open set. denote $\bar{\mathcal{D}}$ its closure.

we call f is analytic on $\bar{\mathcal{D}}$. if there is U s.t.

$\bar{\mathcal{D}} \subset U$. and f is analytic in U.

Thm. if f is analytic in $\bar{\mathcal{D}}$, then f is unique determined by the value of f on the bdry of $\mathcal{D}$.

Thm. Reflection principle



$\mathcal{D}$ is the shaded region. $\mathcal{D}^+$ and $\mathcal{D}_-$ are its upper and lower part, respectively. Assume f is analytic in $\mathcal{D}$. then

$\overline{f(z)} = f(\bar{z}) \iff f$ is real valued on the segment $\mathcal{D} \cap \{x\text{-axis}\}$.