lecture 3:

- Definition of function.

ex: $f(z) = z^2$. $\quad x+iy \longrightarrow x^2-y^2+2xyi$

ex $f(z) = |z|^2$.

ex. polynomial. $\quad P_n(z) = a_0 + \cdots + a_n z^n$

ex. Rational function $\quad R(z) = \dfrac{P(z)}{Q(z)}$

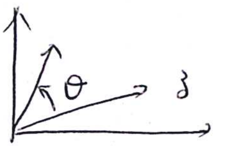
ex. $z^{1/2} = \pm r e^{i\theta/2}$ Here $\theta = \operatorname{Arg}(z)$.

if sign is fixed to be "+". then a function is defined.

ex. Translation

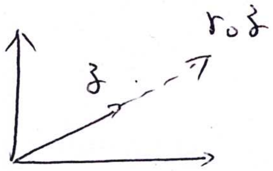
$T(z) = z + a_0$

ex: Rotation



$Rot_\theta(z) = e^{i\theta} z$

ex: Scaling



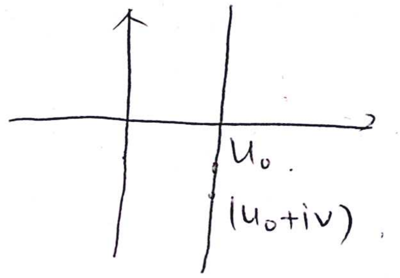
ex:

$$f(z) = a_0 z + b_0 = a_0\left(z + \frac{b_0}{a_0}\right) = r_0 e^{i\theta_0}\left(z + \frac{b_0}{a_0}\right) \quad \text{if } a_0 = r_0 e^{i\theta_0}$$

$$\therefore \quad z \xrightarrow{\text{translation}} z + \frac{b_0}{a_0} \xrightarrow{\text{Rotation}} e^{i\theta_0}\left(z + \frac{b_0}{a_0}\right) \xrightarrow{\text{Scaling}} r_0 e^{i\theta_0}\left(z + \frac{b_0}{a_0}\right)$$

<u>ex:</u> $w = z^2$

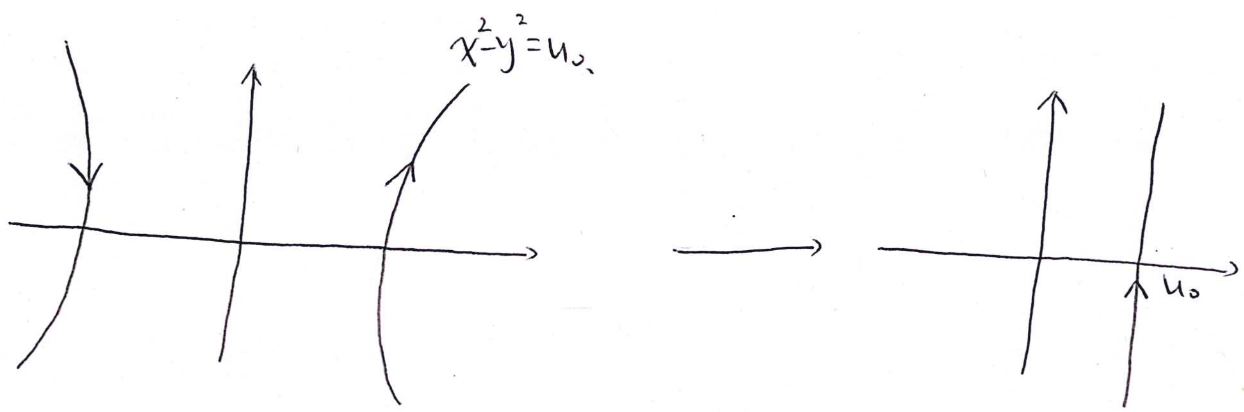
find pre-image of the points on the vertical Line



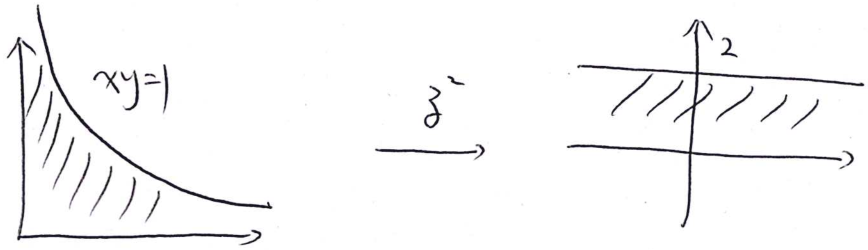
$$\Rightarrow \begin{cases} x^2 - y^2 = u_0 \\ 2xy = v \end{cases} \Rightarrow \begin{cases} x = \frac{\sqrt{2}}{2}\cdot\sqrt{u_0 + \sqrt{u_0^2 + v^2}} \\ y = \dfrac{v}{\sqrt{2}\cdot\sqrt{u_0 + \sqrt{u_0^2+v^2}}} \end{cases}$$

and

$$\begin{cases} x = -\frac{\sqrt{2}}{2}\cdot\sqrt{u_0 + \sqrt{u_0^2 + v^2}} \\ y = -\dfrac{v}{\sqrt{2}\sqrt{u_0 + \sqrt{u_0^2+v^2}}}. \end{cases}$$

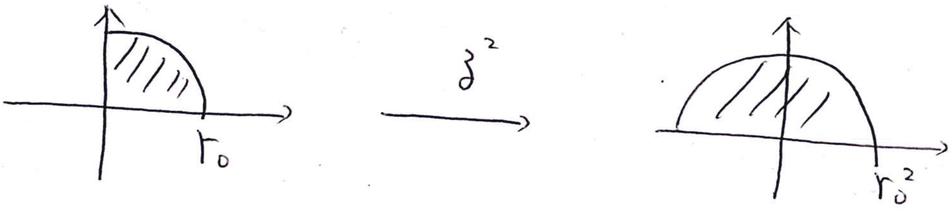


ex:



ex: $re^{i\theta} \xrightarrow{z^2} r^2 e^{i2\theta}$.

argument is doubled.



Limit

① $\lim_{z \to z_0} f(z) = w_0$

$|f(z) - w_0| < \varepsilon$ if $|z - z_0| < \delta(\varepsilon)$

② $\lim_{z \to \infty} f(z) = w_0$

$|f(z) - w_0| < \varepsilon$ if $|z| > M(\varepsilon)$

③ $\lim_{z \to z_0} f(z) = \infty$.

$|f(z)| > M$ if $|z - z_0| < \delta(M)$

ex:

$$\lim_{z \to 1} \frac{i\bar{z}}{2} = \frac{i}{2}.$$

ex:

$$\lim_{z \to 0} \frac{z}{\bar{z}} \text{ does not exist}$$

<u>Rk</u>. Limit if exists does not depends on the way how we approach the limiting pt. if you approach to a same pt by different ways and obtain different limit, then the function has no limit at this point.

Basic Thm:

$$\lim f \pm g = \lim f \pm \lim g$$

$$\lim fg = \lim f \cdot \lim g$$

$$\lim f/g = \lim f / \lim g$$

<u>Continuity</u>.

$$\lim_{z \to z_0} f(z) = f(z_0)$$

then f is cont at $z = z_0$.