

Math4230 Exercise 7

- Find the conjugate function of the following f :
 - $f(x) = -\log x$
 - $f(x) = \frac{1}{2}x^T Qx$, where $Q \in \mathbb{R}^{n \times n}$ is a symmetric positive definite matrix and $x \in \mathbb{R}^n$
- Find the conjugate function of the following functions in terms of g^* , the conjugate function of g .
 - $f_1(x) = g(x) + a^T x + b$
 - $f_2(x) = g(x - b)$
- Let $f(x) = \|x\|$. Show that

$$\partial f(x) = \begin{cases} \{x/\|x|\}, & x \neq 0 \\ \{g \mid \|g\| \leq 1\}, & x = 0 \end{cases}$$

- Let $f : \mathbb{R}^n \rightarrow (-\infty, \infty]$ be a proper convex function. Suppose $g_x \in \partial f(x)$, $g_y \in \partial f(y)$. Show that

$$\langle g_x - g_y, x - y \rangle \geq 0$$