Math4230 Exercise 10

1. Consider the following problem
$$\min x^2 + 1 \text{ subject to } (x-2)(x-4) \le 0$$
   (a) Find the feasible set, optimal value and the optimal solution.
   (b) Write down the Lagrangian $L(x, \lambda)$. Find the dual function q.
   (c) Solve the dual problem. Does strong duality hold?

2. Consider the following problem
$$\min\langle c, x\rangle, \text{ subject to } f(x) \le 0$$
   with $c \ne 0$.
   Express the dual problem in terms of the conjugate function of f.

3. Consider
$$\min x_1^2 + x_2^2$$
$$\text{subject to } (x_1 - 1)^2 + (x_2 - 1)^2 \le 1, \ (x_1 - 1)^2 + (x_2 + 1)^2 \le 1$$
   (a) Find the feasible set, optimal solution x^* and optimal value p^*.
   (b) Write down the KKT conditions. Can you find λ_1^*, λ_2^* such that $x^*, (\lambda_1^*, \lambda_2^*)$ satisfy the KKT conditions?