Course Outline

Optimization Theory (MATH4230)

2021/2022, Second Term: 10 January 2022 (Mon) – 23 April 2022 (Sat)

No Teaching Days:
Jan 31-Feb 5 (Mon-Sat, Lunar New Year Vacation),
April 5, Ching Ming Festival,
April 15-18(Fri-Mon, Easter).

Course Homepage:
http://www.math.cuhk.edu.hk/course/2021/math4230

Lectures:
Lecture: Tue 2:30pm - 4:15pm (Wu Ho Man Yuen 407);
Wed 1:30pm - 2:15pm (Y.C. Liang Hall G04)

Tutorial:
Wed 12:30pm - 1:15pm (Y.C. Liang Hall G04)

Teacher:
Professor Tieyong Zeng

Tutor:
Wong Hok Shing, hswong@math.cuhk.edu.hk

Course Description:

Unconstrained and equality optimization models, constrained problems, optimality conditions for constrained extrema, convex sets and functions, duality in nonlinear convex programming, descent methods, conjugate direction methods and quasi-Newton methods. Students taking this course are expected to have knowledge in advanced calculus.

Subject Content in Outline:

1. Introduction to optimization, example problems
2. Convexity
   - **a.** convex sets
   - **b.** closest point problem and its dual
   - **c.** convex functions
   - **d.** Fenchel duality
3. Unconstrained optimization
   - **a.** basic theory
   - **b.** gradient descent
   - **c.** accelerated first-order methods
   - **d.** Newton's method
   - **e.** quasi-Newton methods

4. Constrained optimization

   a. geometric optimality conditions
   b. KKT conditions
   c. Lagrange duality with examples
   d. interior point methods
   e. ADMM

5. Modeling

   a. applications in engineering, statistics, and machine learning
   b. convex relaxations

6. Non-smooth optimization

   a. subgradients and basic theory
   b. subgradient method
   c. proximal methods
   d. proximal gradient (forward-backward splitting)

Course prerequisite:

Most fundamental: advanced calculus and linear algebra.

The course is focused on both optimization methods and theoretical analysis. The students should be very solid in mathematical analysis, and have a very good feeling and understanding of numerical methods and rigorous mathematical reasoning. It is advised to take at Year 3 or 4.

Grade policies:

Tutorial attendance & good efforts or top 15% in both the mid- and final exams:
10%;

(tutorial assignments are counted only if they are submitted before 6:30pm Monday next after the tutorial class)

Mid-Exam/Project: **35%**; Final Exam: **55%.**

Mid-exam date:

Attention: Venue may be different from the currently used classroom.

Textbooks: mainly based on

1. **S. Boyd and L. Vandenberghe**, *Convex Optimization*, Cambridge University Press, 2004.

2. **D. Bertsekas, A. Nedic, A. Ozdaglar**, *Convex Analysis and Optimization* Athena Scientific, 2003.

3. **D. Bertsekas**, *Convex Optimization Theory*, Athena Scientific, 2009.

4. **Boris S. Mordukhovich, Nguyen Mau Nam**, *An Easy Path to Convex Analysis and Applications*, Morgan & Claypool Publishers, 2013.

5. **D. Bertsekas**, *Convex Optimization Algorithms*, Athena Scientific, 2015.

6. **D. Michael Patriksson**, *An Introduction to Continuous Optimization: Foundations and Fundamental Algorithms*, Third Edition (Dover Books on Mathematics), 2020.

References:

1. **A. Ben-Tal and A. Nemirovski**, *Lectures on Modern Convex Optimization* (SIAM).
2. **J. M. Borwein and A. S. Lewis**, *Convex Analysis and Nonlinear Optimization* (Springer).
3. **J.B. Hiriart-Urruty and C. Lemarechal**, *Convex Analysis and Minimization Algorithms* (Springer).
4. **D. Luenberger and Y. Ye**, *Linear and Nonlinear Programming* (Springer).
5. **Y. Nesterov**, *Introductory Lectures on Convex Optimization: A Basic Course* (Kluwer).
6. **J. Nocedal and S. Wright**, *Numerical Optimization* (Springer).

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http://www.cuhk.edu.hk/policy/academichonesty/