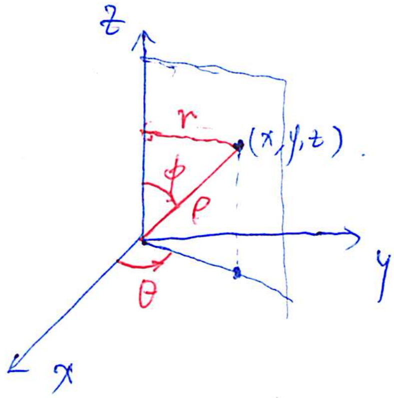
Triple Integral

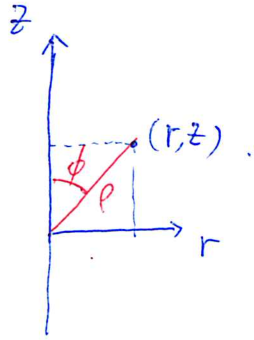
rectangular v.s. cylindrical v.s. Spherical.

$(x,y,z) \longleftrightarrow (r,\theta,z) \longleftrightarrow (\rho,\phi,\theta)$.

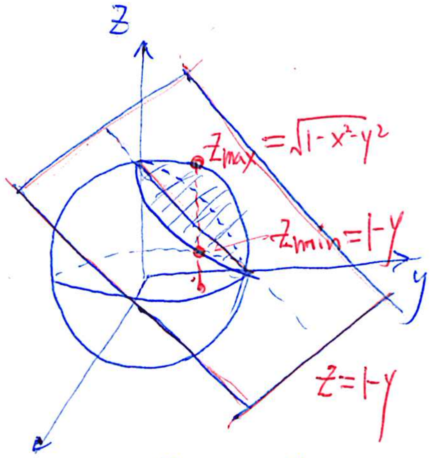
$x = r\cos\theta$, $y = r\sin\theta$; $z = \rho\cos\phi$, $r = \rho\sin\phi$

Volume form $\boxed{dV = dx\,dy\,dz = r\,dr\,d\theta\,dz = \rho^2\sin\phi\,d\rho\,d\phi\,d\theta}$.





Example: Volume of region: $z > 1-y$, inside unit ball.



- Set up in rectangular coordinate:

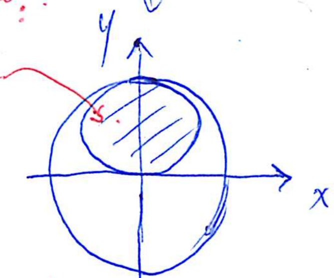
$$\int_0^1 \int_{-\sqrt{2y-2y^2}}^{\sqrt{2y-2y^2}} \int_{1-y}^{\sqrt{1-x^2-y^2}} dz\,dx\,dy$$

Points when plane below sphere:

$1-y \le \sqrt{1-x^2-y^2}$.

$\Longleftrightarrow \quad x^2+2y^2-2y \le 0$.

↓ "Shadow" on x-y



- Rotate and set up in spherical coordinate.

$$\int_0^{2\pi}\int_0^{\frac{\pi}{4}}\int_{\frac{1}{\sqrt{2}\cos\phi}}^{1} \rho^2\sin\phi\,d\rho\,d\phi\,d\theta$$

$$= \frac{2\pi}{3} - \frac{5\pi}{6\sqrt{2}}$$