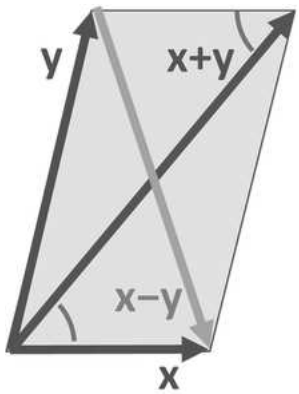
MATH 2010B Advanced Calculus I
(2014-2015, First Term)
Quiz 1
Suggested Solution

1. For any vectors $\mathbf{x} = (x_1, x_2, \cdots, x_n), \mathbf{y} = (y_1, y_2, \cdots, y_n)$ in $\mathbb{R}^n$, we have

$$\begin{aligned} ||\mathbf{x}+\mathbf{y}||^2 + ||\mathbf{x}-\mathbf{y}||^2 &= \sum_{i=1}^n (x_i+y_i)^2 + \sum_{i=1}^n (x_i - y_i)^2 \\ &= \sum_{i=1}^n (x_i+y_i)^2 + (x_i-y_i)^2 \\ &= \sum_{i=1}^n x_i^2 + 2x_iy_i + y_i^2 + x_i^2 - 2x_iy_i + y_i^2 \\ &= \sum_{i=1}^n 2x_i^2 + 2y_i^2 \\ &= 2\sum_{i=1}^n x_i^2 + 2\sum_{i=1}^n y_i^2 \\ &= 2||\mathbf{x}||^2 + 2||\mathbf{y}||^2 \end{aligned}$$

The geometric meaning is illustrated as following graph:



Hence, the geometric meaning is that the sum of the squares of the sides of a parallelogram is the same as the sum the squares of the diagonals

2. We have

$$\begin{cases} 2x + y - 3z &= 3 \ldots\ldots\ldots\ldots(1) \\ x + 2y + 3z &= 6 \ldots\ldots\ldots\ldots(2) \end{cases}$$

Then $(1) + (2) \Rightarrow 3x + 3y = 9 \Rightarrow x + y = 3$.
Let $x = t$, then $y = 3 - t$. Put back into (1), $2(t) + (3 - t) - 3z = 3 \Rightarrow z = \dfrac{t}{3}$.
Therefore, the parametric form of the line L in $\mathbb{R}^3$ is

$$(x, y, z) = (t, 3 - t, \frac{t}{3}) = (0, 3, 0) + t(1, -1, \frac{1}{3}), \quad t \in \mathbb{R}$$

♠ ♡ ♣ ♢ END ♢ ♣ ♡ ♠