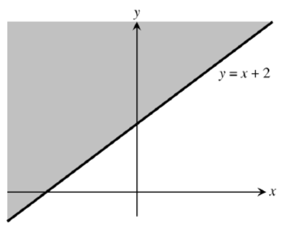
MATH 2010A/B Advanced Calculus I
(2014-2015, First Term)
Homework 4
Suggested Solution

Exercises 14.1

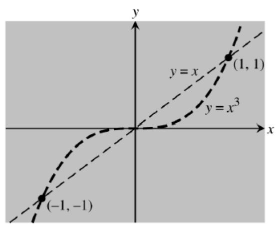
2. (b) $f(-3, \frac{\pi}{12}) = -\frac{1}{\sqrt{2}}$

3. (b) $f(1, \frac{1}{2}, \frac{1}{2}) = \frac{8}{5}$

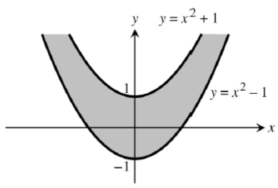
5. Domain: all points (x, y) on or above the line $y = x + 2$



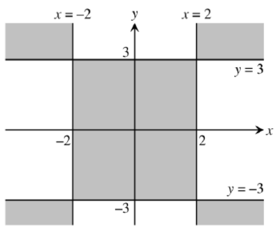
7. Domain: all points (x, y) not lying on the graph of $y = x$ or $y = x^3$



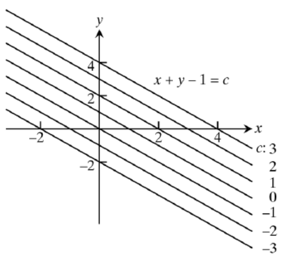
9. Domain: all points (x, y) satisfying $x^2 - 1 \leq y \leq x^2 + 1$



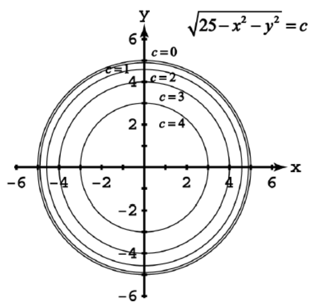
11. Domain: all points (x, y) satisfying $(x-2)(x+2)(y-3)(y+3) \geq 0$



13.



16.

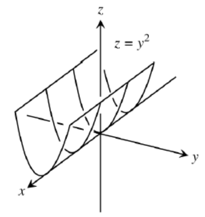


31. f

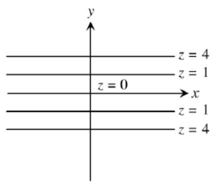
32. e

33. a

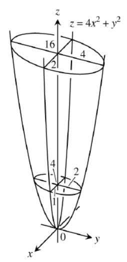
37. (a)



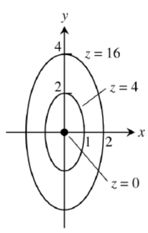
(b)



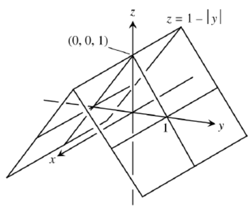
43. (a)



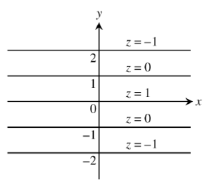
(b)



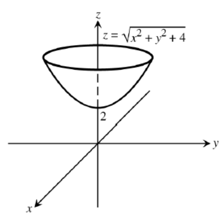
45. (a)


z
z = 1 − |y|
(0, 0, 1)
1
y
x

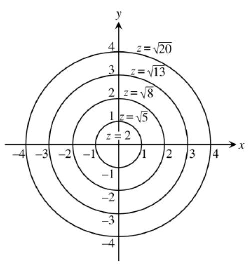
(b)


y
z = −1
z = 0
z = 1
z = 0
z = −1
2
1
0
−1
−2
x

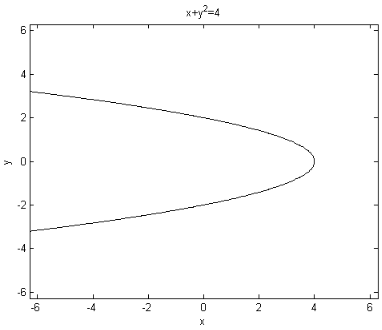
47. (a)


z
z = √(x² + y² + 4)
2
y
x

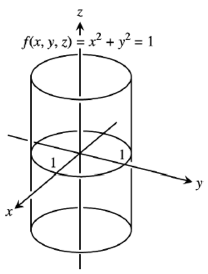
(b)


y
z = √20
z = √13
z = √8
z = √5
z = 2
x

51. $f(x,y) = \sqrt{x + y^2 - 3}$ and pass through the point $(3, -1)$.
Therefore, $z = \sqrt{3 + (-1)^2 - 3} = 1 \Rightarrow x + y^2 - 3 = 1 \Rightarrow x + y^2 = 4$.



57.



64. $g(x,y,z) = \dfrac{x - y + z}{2x + y - z}$ at $(1, 0, -2) \Rightarrow w = \dfrac{x - y + z}{2x + y - z}$.
At $(1, 0, -2) \Rightarrow w = \dfrac{1 - 0 + (-2)}{2(1) + 0 - (-2)} = -\dfrac{1}{4} \Rightarrow -\dfrac{1}{4} = \dfrac{x - y + z}{2x + y - z} \Rightarrow 2x - y + z = 0$.

Exercises 14.2

1. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{3x^2 - y^2 + 5}{x^2 + y^2 + 2} = \frac{3(0)^2 - 0^2 + 5}{0^2 + 0^2 + 2} = \frac{5}{2}$

9. $\displaystyle\lim_{(x,y)\to(0,0)} \frac{e^y \sin x}{x} = \lim_{(x,y)\to(0,0)} e^y \frac{\sin x}{x} = \lim_{(x,y)\to(0,0)} e^y \lim_{(x,y)\to(0,0)} \frac{\sin x}{x} = 1 \cdot 1 = 1$

15. $\displaystyle\lim_{\substack{(x,y)\to(1,1) \\ x\neq 1}} \frac{xy - y - 2x + 2}{x - 1} = \lim_{\substack{(x,y)\to(1,1) \\ x\neq 1}} \frac{(x-1)(y-2)}{x - 1} = \lim_{(x,y)\to(1,1)} y - 2 = (1 - 2) = -1$

17. $\displaystyle\lim_{\substack{(x,y)\to(0,0) \\ x\neq y}} \frac{x - y + 2\sqrt{x} - 2\sqrt{y}}{\sqrt{x} - \sqrt{y}} = \lim_{\substack{(x,y)\to(0,0) \\ x\neq y}} \frac{(\sqrt{x} - \sqrt{y})(\sqrt{x} + \sqrt{y} + 2)}{\sqrt{x} - \sqrt{y}} = \lim_{(x,y)\to(0,0)} (\sqrt{x} +$
$\sqrt{y} + 2) = (\sqrt{0} + \sqrt{0} + 2) = 2$

19. $\displaystyle\lim_{\substack{(x,y)\to(2,0)\\2x-y\neq4}} \frac{\sqrt{2x-y}-2}{2x-y-4} = \lim_{\substack{(x,y)\to(2,0)\\2x-y\neq4}} \frac{\sqrt{2x-y}-2}{(\sqrt{2x-y}-2)(\sqrt{2x-y}+2)} = \lim_{(x,y)\to(2,0)} \frac{1}{\sqrt{2x-y}+2} =$
$\displaystyle\frac{1}{\sqrt{2(2)-0}+2} = \frac{1}{4}$

33. (a) All (x, y)
(b) All (x, y) except $(0, 0)$

39. (a) All (x, y, z) such that $z > x^2 + y^2 + 1$
(b) All (x, y, z) such that $z \neq \sqrt{x^2 + y^2}$

43. $\displaystyle\lim_{\substack{(x,y)\to(0,0)\\\text{along } y=kx^2}} \frac{x^4-y^2}{x^4+y^2} = \lim_{x\to0} \frac{x^4-(kx^2)^2}{x^4+(kx^2)^2} = \lim_{x\to0} \frac{x^4-k^2x^4}{x^4+k^2x^4} = \frac{1-k^2}{1+k^2}$
Therefore, different limits for different values of k

44. $\displaystyle\lim_{\substack{(x,y)\to(0,0)\\\text{along } y=kx,\ k\neq0}} \frac{xy}{|xy|} = \lim_{x\to0} \frac{x(kx)}{|x(kx)|} = \lim_{x\to0} \frac{kx^2}{|kx^2|} = \lim_{x\to0} \frac{k}{|k|}$
The limit is 1 if $k > 0$ and the limit is -1 if $k < 0$. Therefore, it has no limits.

49. $\displaystyle\lim_{\substack{(x,y)\to(1,1)\\\text{along } x=1}} \frac{xy^2-1}{y-1} = \lim_{y\to1} \frac{y^2-1}{y-1} = \lim_{y\to1} (y+1) = 2$
$\displaystyle\lim_{\substack{(x,y)\to(1,1)\\\text{along } y=x}} \frac{xy^2-1}{y-1} = \lim_{y\to1} \frac{y^3-1}{y-1} = \lim_{y\to1} (y^2++y+1) = 3$
Therefore, limits do not exist.

51. (a) $\displaystyle\lim_{(x,y)\to(0,1)} f(x,y) = 1$ since any path through $(0, 1)$ that is close to $(0, 1)$ satisfies $y \geq x^4$
(b) $\displaystyle\lim_{(x,y)\to(2,3)} f(x,y) = 0$ since any path through $(2, 3)$ that is close to $(2, 3)$ does not satisfy $y \geq x^4$ or $y \leq 0$
(c) $\displaystyle\lim_{\substack{(x,y)\to(0,0)\\\text{along } x=0}} f(x,y) = 1$ and $\displaystyle\lim_{\substack{(x,y)\to(0,0)\\\text{along } y=x^8}} f(x,y) = 0$. Therefore, limit does not exist.

52. (a) $\displaystyle\lim_{(x,y)\to(3,-2)} f(x,y) = 3^2 = 9$ since any path through $(3, -2)$ that is close to $(3, -3)$ satisfies $x \geq 0$
(b) $\displaystyle\lim_{(x,y)\to(-2,1)} f(x,y) = (-2)^3 = -8$ since any path through $(-2, 1)$ that is close to $(-2, 1)$ satisfies $x < 0$
(c) $\displaystyle\lim_{(x,y)\to(0,0)} f(x,y) = 0$ since the limit is 0 along any path through $(0, 0)$ with $x < 0$ and the limit is also zero along any path through $(0, 0)$ with $x \geq 0$

57. The limit is 0 since $\left|\sin(\frac{1}{x})\right| \le 1 \Rightarrow -1 \le \sin(\frac{1}{x}) \le 1 \Rightarrow -y \le y\sin(\frac{1}{x}) \le y$ for $y \ge 0$, and $-y \ge y\sin(\frac{1}{x}) \ge y$ for $y \le 0$. Thus as $(x, y) \Rightarrow (0, 0)$, both $-y$ and y approach $0 \Rightarrow y\sin(\frac{1}{x}) \to 0$ by the Sandwich Theorem.

68. $\lim_{(x,y)\to(0,0)} \frac{3xy^2}{x^2+y^2} = \lim_{r\to 0} \frac{(3r\cos\theta)(r^2\sin^2\theta)}{r^2} = \lim_{r\to 0} 3r\cos\theta\sin^2\theta = 0$
Therefore, define $f(0, 0) = 0$, then f is continuous at the origin.

♠ ♡ ♣ ♢ END ♢ ♣ ♡ ♠