THE CHINESE UNIVERSITY OF HONG KONG
Department of Mathematics

MATH 2050C Mathematical Analysis I

Tutorial 5 (February 22)

The following were discussed in the tutorial this week:

1 Monotone Sequences

1.1 Monotone Convergence Theorem. *A monotone sequence of real number is convergent if and only if it is bounded. Furthermore,*

(a) If (x_n) is a bounded increasing sequence, then $\lim(x_n) = \sup\{x_n : n \in \mathbb{N}\}$.

(b) If (y_n) is a bounded decreasing sequence, then $\lim(y_n) = \inf\{y_n : n \in \mathbb{N}\}$.

Example 1.1. *(Calculation of square roots)*

Let $a > 0$. Let $s_1 > 0$ be arbitrary and define $s_{n+1} := \frac{1}{2}(s_n + a/s_n)$ for $n \in \mathbb{N}$. Show that the sequence (s_n) converges to $\sqrt{a}$.

Example 1.2. *Let $x_1 > 1$ and $x_{n+1} := 2 - \frac{1}{x_n}$ for $n \in \mathbb{N}$. Show that (x_n) is bounded and monotone. Find the limit.*

Example 1.3. *Let $x_n := 1/1^2 + 1/2^2 + \cdots + 1/n^2$ for each $n \in \mathbb{N}$. Prove that (x_n) is increasing and bounded, and hence convergent.*

2 Classwork

Let $x_1 := 1$ and $x_{n+1} := \sqrt{2 + x_n}$ for $n \in \mathbb{N}$. Show that (x_n) converges and find the limit.