MATH2050A HW6 Solution

6. Let $\epsilon > 0$ be given. Since g is continuous at b, there exists $\delta > 0$ s.t. for all $|y-b| < \delta$, $|g(y) - g(b)| < \epsilon$. Also, $\lim_{x\to c} f(x) = b$ implies there exists $\delta_x > 0$ s.t. for all $0 < |x-c| < \delta_x$, $|f(x) - b| < \delta$. This implies $|g(f(x)) - g(b)| < \epsilon$. Therefore, $\lim_{x\to c} g \circ f = g(b)$.

10. $c \in P$ implies $f(c) > 0$. Take $\epsilon = f(c)/10 > 0$, by the definition of continuity of f at c, there exists $\delta > 0$ s.t. if $x \in V_\delta(c)$, $|f(x) - f(c)| < f(c)/10$. In particular, $f(x) > 9f(c)/10 > 0$ and $V_\delta(c) \subseteq P$ as desired

    Remark. This question is a rephrase of HW5 question 7.

15. For $f(x) \geq g(x)$, $\frac{1}{2}(f(x)+g(x))+\frac{1}{2}|f(x)-g(x)| = \frac{1}{2}(f(x)+g(x))+\frac{1}{2}(f(x)-g(x)) = f(x) = \sup\{f(x), g(x)\} = h(x)$.

    For $f(x) < g(x)$, $\frac{1}{2}(f(x)+g(x))+\frac{1}{2}|f(x)-g(x)| = \frac{1}{2}(f(x)+g(x))+\frac{1}{2}(g(x)-f(x)) = g(x) = \sup\{f(x), g(x)\} = h(x)$.

    Since f, g are continuous at c, so as $f+g$, $f-g$ and $|f-g|$. Thus h is continous at c.