MATH 2060B - HW 5
Due Date: 10 March 2021, 23:59

Problems: Ex7.3 P.224: 14, 17, 22 (3 Questions in total)

Textbook: Bartle RG, Sherbert DR(2011). Introduction to Real Analysis, fourth edition, John Wiley Sons,Inc.

Instruction:

1. Please submit your solution in one pdf file to Blackboard.
2. Rename your file in the form "HW1_ChanTaiMan_1155151031".
3. You are reminded that your HW is graded based on **both** your idea and your presentation

Questions:

1 (P.224 Q14)**.** Show that there does NOT exist a function $f \in \mathcal{C}^1[0,2]$ (cotinuously differentiable on $[0,2]$) such that

i. $f(0) = -1$

ii. $f(2) = 4$

iii. $f'(x) \leq 2$ for all $x \in [0,2]$

(The Fundamental Theorem may be useful).

2 (P.224 Q17)**.** Let $J := [\alpha, \beta]$ and let $\phi : J \to \mathbb{R}$ be cotinuously differentiable on J. Let $f : I \to R$ be continuous on an interval I such that $\phi(J) \subset I$.
Define $F(u) := \int_{\phi(\alpha)}^{u} f(x)dx$ for all $u \in I$ and $H(t) := F(\phi(t))$ for all $t \in J$.

i. Show that $H'(t) = f(\phi(t))\phi'(t)$ for all $t \in J$

ii. Show that
$$\int_{\phi(\alpha)}^{\phi(\beta)} f(x)dx = F(\phi(\beta)) = H(\beta) = \int_{\alpha}^{\beta} f(\phi(t))\phi'(t)dt$$

Remark. This is a proof to the Substitution Theorem (Theorem 7.3.8)

3 (P.224 Q22)**.** Let $h : [0,1] \to \mathbb{R}$ be the Thomae's function, that is

$$h(x) = \begin{cases} \frac{1}{p} & x = \frac{q}{p}, \gcd(q,p) = 1, q, p \in \mathbb{N} \\ 0 & x \notin \mathbb{Q} \end{cases}$$

for all $x \in [0,1]$. Let sgn be the sign function. Show that the composition $\operatorname{sgn} \circ h$ is not Riemann integrable on $[0,1]$.