MATH 2060B - HW 3
Due Date: 10 Feb 2021, 23:59

Problems: Ex6.3 P.187: 5; Ex6.4 P.196: 10 (2 Questions in total)

Textbook: Bartle RG, Sherbert DR(2011). Introduction to Real Analysis, fourth edition, John Wiley Sons,Inc.

Instruction:

1. Please submit your solution in one pdf file to Blackboard.
2. Rename your file in the form "HW1_ChanTaiMan_1155151031".
3. You are reminded that your HW is graded based on **both** your idea and your presentation

Questions:

1 (P.187 Q5)**.** Let $f(x) := \begin{cases} x^2 \sin(1/x) & x \neq 0 \\ 0 & x = 0 \end{cases}$ and let $g(x) := \sin(x)$ for all $x \in \mathbb{R}$.

a. Show that $\lim_{x\to 0} \frac{f(x)}{g(x)} = 0$

b. Show that $\lim_{x\to 0} \frac{f'(x)}{g'(x)}$ does not exist.

2 (P.196 Q10)**.** Let $h(x) := \begin{cases} e^{-1/x^2} & x \neq 0 \\ 0 & x = 0 \end{cases}$ for all $x \in \mathbb{R}$.

a. Show that $h^{(n)}(0) = 0$ for all $n \in \mathbb{N}$.

b. Suppose $x \neq 0$. Show that the remainder term obtained by applying the Taylor's Theorem to the points $x, x_0 := 0$ and h as an $n-$ times differentiable function does not converge to 0 as $n \to \infty$

(If you have spent enough efforts but without progress, you may consult the hint in the footnote[1].)

[1] Hint: Try to first show that $\lim_{x\to 0} h(x)/x^k = 0$ for all $k \in \mathbb{N}$ by the L'Hospital Rule. The Leibniz's Rule, or the high-order product rule, may be useful to compute $h^n(x)$ for $x \neq 0$ and $n \in \mathbb{N}$ in the process: let $f, g : I \to \mathbb{R}$ be functions defined on an open interval I and $n \in \mathbb{N}$. The Leibniz's Rule states that if f, g are n-times differentiable at $x \in I$, then the derivative of the product at x can be computed by $(fg)^{(n)}(x) = \sum_{k=0}^{n} \binom{n}{k} f^{(n-k)}(x) g^k(x)$.