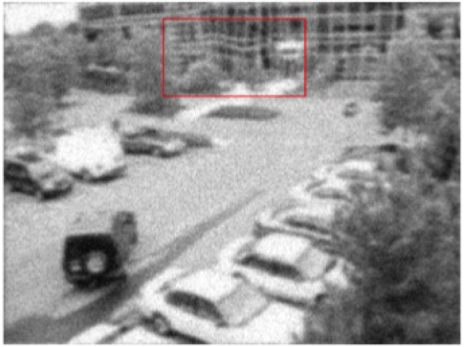
Lecture 11: Image deblurring



Atmospheric turbulence

Motion Blur

Speeding problem

Image deblurring in the frequency domain:

Mathematical formulation of image blurring

Let g be the observed (blurry) image.

Let f be the original (good) image.

Model g as: $g = H(f) + n$

where H is the degradation function/operator and n is the additive noise.

Assumption on H:

1. H is position invariant:

   Let $g(x,y) = H(f)(x,y)$ and let $\tilde{f}(x,y) := f(x-\alpha, y-\beta)$.

   Then: $H(\tilde{f})(x,y) = g(x-\alpha, y-\beta) = H(f)(x-\alpha, y-\beta)$

2. Linear: $H(f_1 + f_2) = H(f_1) + H(f_2)$

   $H(\alpha f) = \alpha H(f)$ where α is a scalar multiplication.

3. Linearity can be extended to integral:

$$H\left(\iint \alpha(u,v) f(x-u, y-v)\, du\, dv\right) = \iint \alpha(u,v) H(f)(x-u, y-v)\, du\, dv$$

With the above assumption, consider an impluse signal:

$$\delta(x,y) = \begin{cases} 1 & \text{if } (x,y) = (0,0) \\ 0 & \text{if } (x,y) \neq (0,0) \end{cases}$$

Then: $f(x,y) = f * \delta(x,y) = \displaystyle\sum_{\alpha=-M/2}^{M/2-1} \sum_{\beta=-N/2}^{N/2-1} f(\alpha,\beta)\,\delta(x-\alpha, y-\beta)$

$$\therefore \quad g(x,y) = H(f)(x,y)$$

$$= \sum_{\alpha=-M/2}^{M/2-1} \sum_{\beta=-N/2}^{N/2-1} f(\alpha,\beta)\, H(\delta)\,(x-\alpha, y-\beta) \quad \text{(by linearity and position-invariant)}$$

$$= \sum_{\alpha=-M/2}^{M/2-1} \sum_{\beta=-N/2}^{N/2-1} f(\alpha,\beta)\, h(x-\alpha, y-\beta) \quad \text{where} \quad h(x,y) = H(\delta)(x,y)$$

$$= f * h\,(x,y)$$

$\therefore$ With the above assumption,

Degradation / Blur = Convolution

Remark:

1. h is called the point spread function

2. $\therefore g(x,y) = h * f(x,y) + n(x,y)$

In the frequency domain,

$$G(u,v) = c\, H(u,v)\, F(u,v) + N(u,v)$$

↑ constant

$\therefore$ Deblurring can be done by:

Compute: $F(u,v) \approx \dfrac{G(u,v)}{cH(u,v)}$ — from observed image — from known degradation

Obtain: $f(x,y) = DFT^{-1}(F(u,v))$

(Does NOT work very well due to noise!)