

**THE CHINESE UNIVERSITY OF HONG KONG**  
**Department of Mathematics**  
**MMAT 5120 (2021-22, Term 2)**  
**Topics in Geometry**  
**Quiz 1**  
**10th February 2022, 8pm - 10pm**

We always denote by  $i$  the imaginary unit  $\sqrt{-1}$ .

1. Find *both* square roots of the following complex numbers
  - (a)  $-9$ ,
  - (b)  $-2i$ ,
  - (c)  $-1 - \sqrt{3}i$ .
  
2. Let  $T : \mathbb{C} \rightarrow \mathbb{C}$  be the geometric transformation given by rotating by  $45^\circ$  about the point  $1 + i$ .
  - (a) Write down a formula for  $T$ .
  - (b) Let  $\alpha, \beta \in \mathbb{C}$  be complex constants. Show that the equation  $\text{Im}(\alpha z + \beta) = 0$ , where  $z$  is a complex variable, defines a straight line in the complex plane  $\mathbb{C}$ .
  - (c) Compute the image of the straight line in part (b) under the transformation  $T$ .
  
3. For any  $\theta \in \mathbb{R}$ , denote by  $R_\theta : \mathbb{C} \rightarrow \mathbb{C}$  the rotation by angle  $\theta$  in the counterclockwise direction around the origin  $0 \in \mathbb{C}$  defined by  $R_\theta(z) = e^{i\theta} \cdot z$  for all  $z \in \mathbb{C}$ .
  - (a) Prove that the set  $G := \{R_\theta : \theta \in \mathbb{R}\}$  of all these rotations is a transformation group.
  - (b) Show that the function  $f : z \mapsto |z|$  is invariant in this geometry.