Department of Mathematics 數學系
The Chinese University of Hong Kong 香港中文大學

The Croucher Foundation Advanced Study Institute
Recent Development in Nonlinear Partial Differential Equations: Part I

Date: 4 March 2011
Venue: Rm. 501a, Academic Building 1, IMS, CUHK

Time / Date	4 March 2011 (Friday)
09:00am – 10:15am	**Professor Michael J. Ward** *University of British Columbia* Title: Traps, Patches, Defects, and Spots: An Asymptotic Analysis of Localized Solutions to Some Diffusive and Reaction-Diffusion Systems (Part II)
10:15am – 10:45am	**Tea Break**
10:45am – 12:00pm	**Professor Michael J. Ward** *University of British Columbia* Title: Traps, Patches, Defects, and Spots: An Asymptotic Analysis of Localized Solutions to Some Diffusive and Reaction-Diffusion Systems (Part III)
12:00pm – 02:00pm	**Working Lunch#**
02:00pm – 02:45pm	**Professor Zongming Guo** *Henan Normal University* Title: Morse index and solution branches of some supercritical problems
02:45pm – 03:30pm	**Professor Fridolin Ting** *Lakehead University* Title: Non-radial finite-energy magnetic vortex solutions to the Ginzburg-Landau equations
03:30pm – 04:00pm	**Tea Break**
04:00pm – 04:45pm	**Professor Zhi-An Wang** *Hong Kong Polytechnic University* Title: Traveling waves of bacterial chemotaxis with chemical diffusion
04:45pm – 05:30pm	**Professor Li Ma** *Henan Normal University* Title: Liouville type theorems for Lichnerowicz equations and Ginzburg-Landau
06:00pm – 08:00pm	**Free**

 # For invited speakers and invited guests only.

Traps, Patches, Defects, and Spots: An Asymptotic Analysis of Localized Solutions to Some Diffusive and Reaction-Diffusion Systems

Professor Michael J. Ward
Department of Mathematics, University of British Columbia

Abstract

A survey of the development of a unified singular perturbation methodology to analyze some linear and nonlinear PDE models of diffusion and reaction-diffusion type with localized solutions is presented. Specific results from this theory are given for four diverse applications.

The first problem is to determine the mean first passage time (MFPT) for free diffusion from within a sphere to small localized traps on its boundary. In the context of cellular signal transduction, the results predict the time-scale needed for a diffusing molecule to arrive at localized signalling compartments on the boundary of a biological cell. From a mathematical viewpoint, the problem of optimizing this MFPT is shown to be closely related to the well-known problem of finding the minimum energy configuration of repelling point charges on the surface of a sphere.

Secondly, in the context of spatial ecology, a long-standing problem is to determine the persistence threshold for extinction of a species in a heterogeneous spatial landscape consisting of either favorable or unfavorable local habitats. For a 2-D spatial landscape consisting of such localized patches, and in the context of the diffusive logistic model, this extinction threshold is calculated asymptotically and the effects of both habitat fragmentation and habitat location on the persistence threshold are obtained. From a mathematical viewpoint, the persistence threshold represents the principal eigenvalue of an indefinite weight singularly perturbed eigenvalue problem.

Thirdly, the dynamics, stability, and self-replication behavior of localized spot-type solutions to the well-known Gray-Scott reaction-diffusion model of chemical physics in a two-dimensional domain are discussed. Reduced ODE systems for the dynamics of spots are given together with phase diagrams in parameter space classifying the different types of spot instabilities.

Finally, in a bounded 2-D domain, we construct solutions exhibiting point-concentration behavior for a fourth order nonlinear eigenvalue problem for the Biharmonic operator with inverse square law nonlinearity. This problem models the deflection of a micro-plate in due to a voltage bias between the deflectable surface and a rigid ground plate. These nearly singular solutions characterize those portions of the bifurcation diagram for which the nonlinearity is, essentially, localized in the domain.

Morse index and solution branches of some supercritical problems

Professor Zongming Guo
Department of Mathematics, Henan Normal University

Abstract

We will study the structure of the global solution branches of some semilinear elliptic problems with supercritical exponents via Morse index. This is a joint work with Professor J. Wei.

Non-radial finite-energy magnetic vortex solutions to the Ginzburg-Landau equations

Professor Fridolin Ting
Department of Mathematical Sciences, Lakehead University

Abstract

We show that there exists non-radial finite-energy degree-changing magnetic vortex solutions to the Ginzburg-Landau equations on all of R^2. We construct non-radial finite-energy degree-changing solutions in the spirit of the construction of non-radial finite-energy sign-changing solutions to the non-linear Schrodinger equation in R^n by Musso, Pacard and Wei. We use the crucial results of non-degeneracy and effective interaction of magnetic vortices derived by Gustafson and Sigal. This is joint work with J. Wei.

Traveling waves of bacterial chemotaxis with chemical diffusion

Professor Zhi-An Wang
Department of Applied Mathematics, Hong Kong Polytechnic University

Abstract

The first mathematical model describing the bacteria chemotaxis was proposed by Keller and Segel on 1971. As a cornerstone of chemotaxis modeling, the Keller-Segel model has become one of the most extensively and persistently studied mathematical models over the past four decades. To simplify the analysis, most of studies on the traveling wave solutions of the Keller-Segel model made a strong assumption that the chemical signal is not diffusible (i.e. the chemical diffusion coefficient is zero). In this talk, I will report some new results of the traveling wave solutions to the full Keller-Segel model with non-zero chemical diffusion. Particularly I will discuss how to connect the Keller-Segel model with Fisher equation and Emden-Fowler equation, and how to derive the singular diffusion limits of traveling wave solutions.

Liouville type theorems for Lichnerowicz equations and Ginzburg-Landau equation

Professor Li Ma
Department of Mathematics, Henan Normal University

Abstract

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In this talk, we firstly review some results of Lichnerowicz equation and Ginzburg-Landau equations. We then discuss the uniform bounds for both equations in R^n. In the last part of this report, we consider the Liouville type theorems for Lichnerowicz equation and Ginzburg-Landau equations in R^n via two approaches from the use of maximum principle and the monotonicity formula.

We remark that the Liouville type theorems are equivalent to a local uniform bound of solutions for a large class of elliptic equations/systems (also for parabolic equation/system). We would like to thank Prof. H.Brezis, who (see also [1]) has informed me that in the statements of Theorems 1 and 2 in [2], the power $p > 1$ should be $p > 2$. Actually, we have used $p > 2$ in the proof of Theorem 1 in [2].

References

[1] H.Brezis, *Comments on two notes by L. Ma and X. Xu*, to appear in C.R.Mathematique, 2011.

[2] Li Ma, *Liouville type theorem and uniform bound for the Lichnerowicz equation and the Ginzburg-Landau equation*,C. R. Acad. Sci. Paris, Ser. I 348 (2010) 993-996